Chapter 6

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Statistics—Risk *and* Return

After reading this chapter you will understand the following:

1. The definition of risk and the various types of risk.
2. The statistical formulas used to measure risk.
3. The definition of return and the various types of return.
4. The difference between systematic and unsystematic risk.
5. The purpose of the Security Market Line (SML) and the Capital Asset Pricing Model.

6.1 Risk

risk

The uncertainty of experiencing a lost. In finance, risk is the uncertainty of a financial loss.

systematic/non-diversifiable risk

Systematic risk is also called non-diversifiable risk because diversification does not eliminate this type of risk. Systematic risk is market or macroeconomic risk.

unsystematic/ diversifiable risk

company or industry specific risk. Diversification can eliminate this type of risk.

risk tolerance

The ability to accept risk at different levels

Risk is defined as the uncertainty of experiencing a loss. In finance, risk is the uncertainty of a financial loss. When you purchase insurance you are insured against a pure risk. A pure risk is a risk where there is a chance of loss or no loss. There is no chance of gain like there is in a speculative risk. There are two primary types of risk that can affect an investment which are systematic risk and unsystematic risk. **Systematic risk** is also called non-diversifiable risk because diversification does not eliminate this type of risk. This means that even if you spread your money across difference asset classes that are negatively correlated, this would not help. On the other hand, diversification is useful in hedging against unsystematic or company specific risk. In simple terms, **unsystematic risk** is company or industry specific, while systematic risk is market or macroeconomic risk. If company X went on strike; the strike would only impact their company or the industry which is unsystematic risk. If another terrorist attack happens like the one on September 11, 2001, the entire markets will be affected. The two types of risk will be defined further later in this chapter.

6.2 Risk Tolerance

In economics there is a belief that individuals would behave in a rational manner. It is rational to act in manner that may ensure a good level of certainty. By nature, most investors are risk averse which means that they prefer to make investment decisions that offer a high level or certainty which means less risk. If a risk averse investor takes on a higher level of risk, they must be compensated with a higher return. Risk averse investors invest in money market mutual funds, or certificate of deposits. Individuals who are risk seekers are on the opposite end of the spectrum when compared to individuals who are risk averse. Risk seeks play the state lottery, bet on horse races and participate in sports like sky diving or extreme skateboarding. In investing, risk seekers pick investments with the highest level of risk, for a chance at a high return. Even if there is a history of low returns, risk seekers will still invest in volatile or risky assets. Risk neutral investors are in the middle of the two extremes. Risk neutral investors are indifferent to the level of risk and are only concerned with their desired rate of return.

6.3 Statistics Overview

One may wonder.... what is statistics and how can studying statistics help in a given profession? Well, almost daily we are exposed to statistics. In studying statistics we must first understand the different types of data, because without data, there can be no statistics. Data consists of information coming from observations, counts, measurements, or responses. Data can be classified as either qualitative or quantitative. Qualitative data can be grouped into specific categories and are referred to as categorical data. Additionally, categorical data are classified as either the nominal or ordinal in their level of measurement. Quantitative data uses numeric values, which can be either continuous or discrete, and are classified as either the interval or ratio in their level of measurement. We'll further discuss these concepts in the coming paragraphs.

The field of statistics can be divided into two major branches: descriptive statistics and inferential statistics. In both branches, we work with a set of measurements in which we are organizing, summarizing, and describing the data. For situations in which data description is our major objective, the set of measurements available to us is frequently the entire population. Good descriptive statistics enable us to make sense of the data by reducing a large set of measurements to a few summary measures that provide a good, rough picture of the original measurements. When we are unable to work with the entire population, we can sample the population and make appropriate measurements, using the results to draw conclusions (inference) about the population. However, for these inferences to be valid, we need to ensure the sample drawn from the population has been collected using random methods.

Think about how statements such as these affect our decision-making?

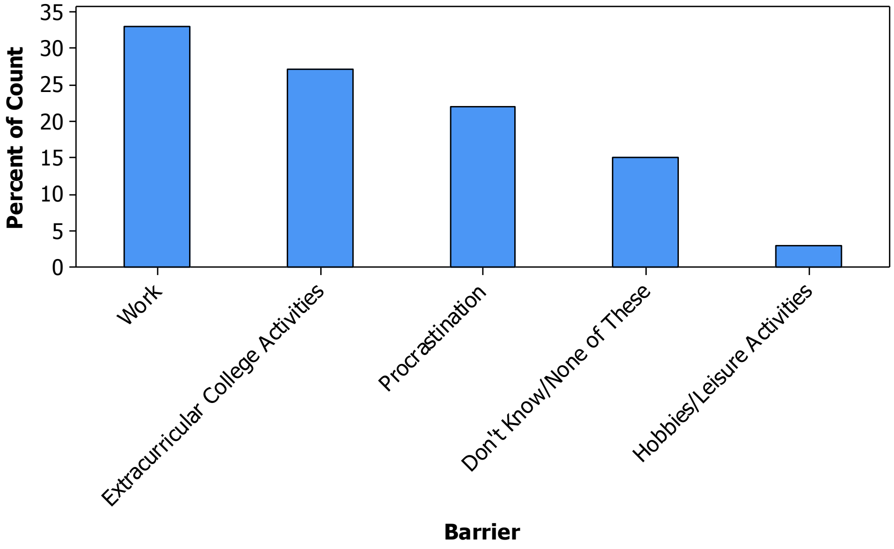
- There's a 75% chance of rain today?
- There is a 25% chance of surviving an operation.
- There is a 1 in 175 million chance of winning the Texas State Lottery.

Sometimes data are presented graphically. If you have ever read the Wall Street Journal, USA Today, or looked at the stock market you have certainly seen SNAPSHOTS that present information in a way that is easy to understand.

Example:

What is the University of Mary Hardin-Baylor (UMHB), college students' biggest barrier to studying for exams?

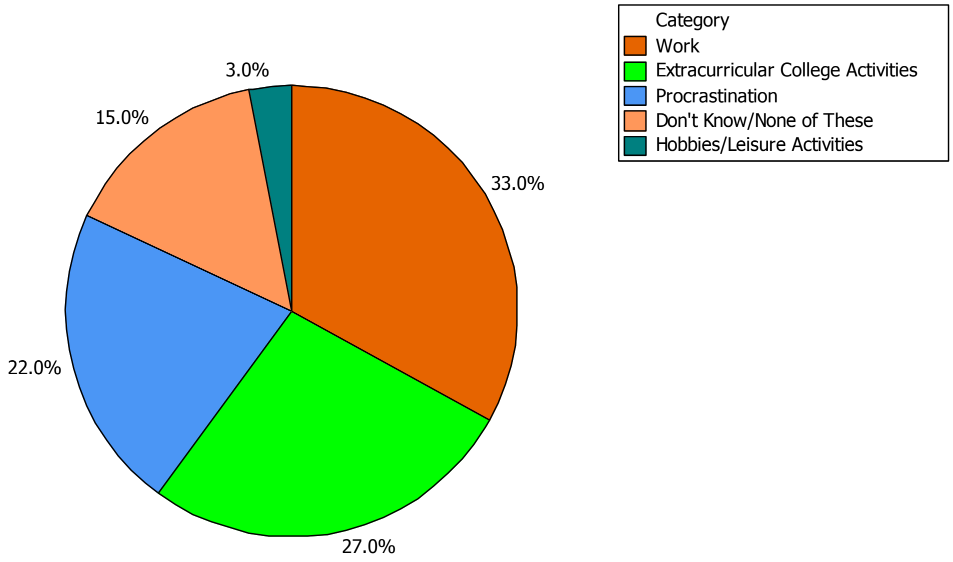
FIGURE 6.1 BAR GRAPH FOR THE BARRIERS TO STUDYING FOR EXAMS AT UMHB



Percent within all data.

Figure 6.1 shows a bar graph of the barriers, where, for UMHB students, the largest barrier is that the student holds either a full time or part time job and is working in addition to his or her studies. Another way of representing the same data is shown in Figure 6.2 as a pie chart.

FIGURE 6.2 PIE CHART FOR THE BARRIERS TO STUDYING FOR EXAMS AT UMHB



In today's environment anyone can access a crazy amount of statistical information. The most successful managers and decision makers actually understand the information and know how to use it effectively. Data are the facts and figures collected, analyzed, and summarized for presentation,

In today's environment anyone can access a crazy amount of statistical information. The most successful managers and decision makers actually understand the information and know how to use it effectively. Data are the facts and figures collected, analyzed, and summarized for presentation, interpretation, and inference. All the data collected in a particular study are referred to as the data set. There are two types of data sets used when studying statistics. These data sets are called populations and samples. A population is the collection of all outcomes, responses, measurements, or counts that are of interest. A sample is a subset of the population.

Example 1: Hypothetical: In a recent survey, in which all 5,500 University of Mary Hardin-Baylor students in Belton Texas were targeted, 1200 students were asked if they read news on the Internet at least once a week. Of the 1,200 students, 700 students responded. Identify the population and the sample. Describe the data set.

Solution: The population consists of all 5,500 UMHB students in Belton Texas, and the sample consists of the 1,200 students at UMHB who were asked to respond to the survey. The data set consists of the responses from the 700 students. Whether a data set is a population or a sample usually depends on the context of the real-life situation. For instance, in this example the population was the set of responses of all college students attending UMHB. Depending on the purpose of the survey, the population could have been the set of responses of all adults who live in California or who have telephones or who read a particular newspaper. When doing a study, it is important to know the kind of data involved. The nature of the data you are working with will determine which statistical procedures that can be used. In most data collection efforts, samples of the population are surveyed because the costs to survey the entire population, which is called a census, are prohibitive Most of the time a sample is taken from a population. The U.S. Census is a good example of when an entire population is surveyed.

Other descriptive statistics include the measures of central tendency (mean, median, and mode), and measures of dispersion or variation (range, variance, standard deviation). Additionally, we can measure dispersion when calculating box and whisker plots, using the interquartile range. Another descriptive statistic is the determination of the relationship between two or more variables by conducting a correlation.

6.3a Types of and Levels of Data

Data sets can consist of two types of data:

- Qualitative data—ranked or unranked categorical factors, i.e. gender, color, level of satisfaction.
- Quantitative data—numerical measures, termed continuous or countable factors, termed discrete, i.e. costs, distance, miles per gallon, or the number of students in your classroom.
    - ✧ Continuous data—numerical values that have the characteristics of real numbers
    - ✧ Discrete data—countable numerical values that have the characteristics of positive whole numbers, including zero.

Another characteristic of data is its scale or level of measurement. The scale or level of measurement determines which statistical calculations are meaningful. The four scales or levels of measurement, in order from lowest to highest, are nominal, ordinal, interval, and ratio.

6.3b Nominal Level of Data (Qualitative Data)

Data at the nominal level of measurement are unranked categorical factors, such as gender, zip codes, type of automobile. Data at this level are categorized using names, label, or qualities. Mathematical computations made at this level are primarily in the form of percentages and frequencies. Nominal level data are generally used by statisticians to define subpopulation within a population. For instance, in the UMHB student survey example, the population of students could be subdivided in terms of gender, race, or religion.

6.3c Ordinal Level of Data (Qualitative Data)

Data at the ordinal level of measurement are ranked categorical factors. Data at this level can be arranged in a meaningful hierarchy, but differences between data entries are not meaningful. Data of this type, in the case of the UMHB survey, may include students' level of satisfaction, for example Dissatisfied, Somewhat Dissatisfied, Neutral, Somewhat Satisfied, and Satisfied. Additionally, ordinal data would look at the level of education of the UMHB student, for example High School, Baccalaureate, Master's, Doctorate, or perhaps the age group of the female students: 17–23, 24–30, 31–36, >36.

6.3d Interval Level of Data (Quantitative Data)

Data at the interval level of measurement can be either continuous or discrete. The data are ordered, and you can calculate the meaningful differences between data entries. At the interval level, a zero entry simply represents a position on a scale; the entry is not an absolute value. Therefore, the interval level of data may be positive or negative real numbers. Examples of continuous data on the interval scale include temperature, where there can be positive and negative temperatures. For instance, a temperature of 0 degrees Celsius does not represent a condition in which no heat is present, it simply means it is really, really cold, and gets colder as the temperature dips into the negative numbers. Other continuous interval data could be altitude because all altitudes are measured from the base sea level, which is 0 feet. Examples of discrete interval data include ladies dress sizes, shoe sizes, and intelligence quotient (IQ) test results. Each example of discrete interval data are whole numbers, but do not have a defined zero point. For example, in ladies dress sizes, there can be a size 0 or a size 00, but this doesn't mean there is an absence of clothing, it only means a very small or petite woman. As in the case of shoe sizes, you may only have sizes in the form of whole or half sizes; and in terms of IQ, test results, which range from 40–175 and can be ranked as shown in Wechsler's Classification in Table 6.1

TABLE 6.1 WECHSLER'S INTELLIGENCE QUOTIENT CLASSIFICATION

Wechsler's Classification		
IQ Range ("Deviation IQ")	**Percentile Rank**	**Intelligence Classification**
130 and over	98 to 99.99	Very superior
120–129	91 to 97	Superior
110–119	75 to 90	High Average
90–109	25 to 73	Average
80–89	9 to 23	Low Average
70–79	2 to 8	Borderline
69 and under	.01 to 2	Extremely Low

6.3e Ratio Level of Data (Quantitative Data)

Data at the ratio level of measurement are similar to data at the interval level, in which the data can be either continuous or discrete, with the added property that a zero is an absolute value, meaning when a measure has a value of zero, that value means nothing or the absence of whatever is being measured. Examples of continuous data at the ratio level include costs, height, weight, length, distance, miles per gallon, and ratios of continuous data. Examples of discrete data at the ratio level include the number of any item that one can count, i.e. the number of cars a parking lot, the number of blue cars in that parking lot, the number of defects in an assembly in a manufacturing plant, or the number of financial portfolios assessed at a given future value.

6.4 Statistical Measures of Risk

6.4a Interval Measure of Central Tendency

The most common numerical descriptive statistic is the average (or mean). The mean is used to measure of central tendency of a population or sample. Central tendency refers to the alignment of all values with respect to the middle value or a typical value of the data and is measured using the mean, median and mode. The most important measure of location is the **mean** or **average** value for a variable. The mean of the data set is the sum of the data entries divided by the number of entries. Simply add up a set of scores and divide by the number of scores. This mean is the first and perhaps most basic statistical formula. The **median** of a data set is the value that lies in the middle value when the odd numbered data set is ordered from lowest to highest, or the average of the two middle values when the even numbered data set is ordered from lowest to highest. The **mode** of a data set is the data entry that occurs with the greatest frequency. If no entry value is repeated, the data set has no mode. A data set or distribution with a single mode is considered unimodal, where the mode is located at the center of the distribution with the highest frequency. Distributions with more than one measurement that occurs with the highest frequency are considered multimodal, of which we might encounter bimodal, trimodal, and so on, distributions.

6.4b Interval Measures of Dispersion

Most times it is important to consider measures of variability or dispersion. For example….suppose that you are a purchasing agent for a large manufacturing firm and you regularly place orders with two different suppliers. After several months of operations you find the mean number of days required to fill orders is 10 days for both suppliers. Although the mean number of days is 10 for both suppliers, the question is do the two suppliers demonstrate the same degree of reliability in terms of making delivery. Receiving materials and supplies on schedule is important. If one of the suppliers has a slower delivery turn around it could be disastrous for a company. This is a situation in which the variability in delivery times may decide on which supplier would be used. The simplest measure of variability or dispersion is the **range**. The range of a data set is the difference between the maximum and minimum data values. Range = (maximum data value) – (minimum data value). Other measures of dispersion include the **variance, standard deviation** and **coefficient of variation.** The variance is the average of the squared deviations from the mean. It is a measure of how far each number or value in the data set is from the mean. The standard deviation is also a measure of how spread the values are from the mean; however, unlike the variance, the standard deviation is in the units of the mean, thus allowing for a simpler comparisons of like populations or samples. In terms of risk, the greater the standard deviation (or variance), the greater the degree of risk. Using the standard deviation gives a "standard" way of knowing what is normal, and what is unusual or very unusual. Data values that are more than two standard deviations from the mean are considered unusual. Data values that are more than three standard deviations from the mean are very unusual. The formula for standard deviation is easy; it is just the square root of the variance.

The coefficient of variation (CV) is a relative measure of the standard deviation in relationship to the magnitude of the mean. The CV is a unit-free number because the standard deviation and mean are measured using the same units. Hence, the CV is often used as an index of process or population variability. In many applications, the CV is expressed as a percentage and is used to compare the variation of two considerably different processes or populations and is defined as the standard deviation divided by the absolute value of the mean, multiplied by 100%. The CV is often used as a measure of risk because it measures the variation of the returns (the standard deviation) relative to the size of the mean return.

As an example, suppose the mean annual return for a particular stock fund (SF1) is 10.39% with a standard deviation of 16.18% and the mean annual return for a different fund (SF2) is 7.7% with a standard deviation of 13.82%. The CV for SF1 then is (16.18/10.39)*100% = 155.73% and the CV for SF2 is (13.82/7.7)*100% = 179.48%. This tells us that for SF1, the standard deviation is 155.73% of its mean annual return and for SF2, the standard deviation is 179.48% of its mean annual return. In comparing the two funds, we see that SF1 has a higher standard deviation than SF2, thus we may have concluded that SF1 had the higher variation, and thus have more risk. However, when taking in account the standard deviation relative to the mean for each stock fund, we actually see that SF2 has the higher coefficient of variation (179.48%), and thus is a riskier investment than SF1. The formulae for calculating the arithmetic mean range, variance, standard deviation and coefficient of variation are given in Table 6.2:

TABLE 6.2 MEASURES OF DISPERSION: FORMULA METHOD

Desired Value	Formula	Key
Range	Range = (MaxVal – MinVal)	MaxVal = Maximum Value MinVal = Minimum Value
Population Variance	$\sigma^2 = \frac{\sum(x-\mu)^2}{N}$	σ^2 = population variance x = value μ = population mean or average N = quantity of values in a population $\sum$ = summation across the values
Sample Variance	$s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$	S^2 = sample variance x = value x = mean or average n = quantity of numbers in the sample $\sum$ = summation across the values
Population Standard Deviation	$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{N}}$	σ = population standard deviation x = value μ = population mean or average N = number of values in population $\sum$ = summation across the values
Sample Standard Deviation	$s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$	s = sample standard deviation x = value $\bar{x} = \mu$ = sample mean or average n = number of values in sample $\sum$ = summation across the values
Coefficient of Variation	$CV = \frac{\sigma}{\lvert\mu\rvert} \times 100\%$ or $CV = \frac{\sigma}{\lvert\bar{x}\rvert} \times 100\%$	σ = population standard deviation $\lvert\mu\rvert$ = absolute value of the population mean or expected rate of return s = sample standard deviation $\bar{x}$ = sample mean or expected rate of return

6.4c Steps to Find the Standard Deviation

- Determine the mean.
- Determine the squared deviations for each value by subtracting the mean from each value, squaring the results for each deviation.
- Add all squared deviations and then divide by either the population size or sample size minus one, depending on whether you are working with a population or a sample.

- Determine the square root of the average of the squared deviations.

Example 1: A sample of MoLamb's returns for the past three years were: –10%, 10%, and 30%. Calculate the range, arithmetic mean, variance, standard deviation and coefficient of variation.

Answer:
Range: [Max – Min] = 30 –(–10) = 40%

Mean: $\bar{x} = \frac{\sum x}{n}$ = (–10 + 10 + 30)/3 = 10%

Variance: $s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$ [(–10 – 10)^2 + (10 – 10)^2 + (30 – 10)^2]/3–1 = 400

Standard Deviation: $s = \sqrt{s^2} = \sqrt{400}$ = 20%

Coefficient of Variation: CV = $\frac{s}{|\bar{x}|} \times$ 100% = 20/10*100% = 200%

Example 2: A sample of MoLamb's returns for the past three years were: 8%, 12%, and 10%. Calculate the range, arithmetic mean, variance, standard deviation and coefficient of variation.

Answer:
Range: [Max – Min] = 10 – 8 = 2%

Mean: $\bar{x} = \frac{\sum x}{n} =$ (8+12+10)/3 = 10%

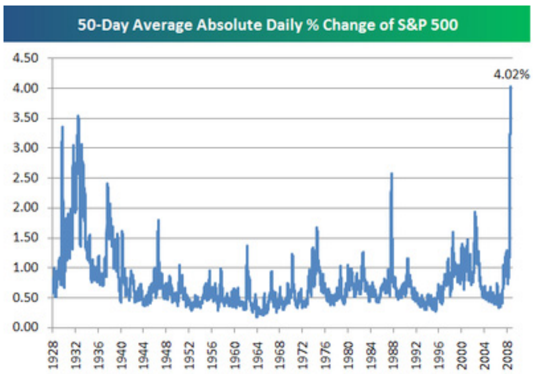
Variance: $s^2 = \frac{\sum (x-\bar{x})^2}{n-1}$ = [(8–10)^2 + (12–10)^2 + (10–10)^2]/(3–1) = 4

Standard Deviation: $s = \sqrt{s^2} = \sqrt{4}$ = 2%

Coefficient of Variation: CV = $\frac{s}{|\bar{x}|} \times$ 100% = 2/10*100% = 20%

6.5 Return

FIGURE 6.3 S&P 500 50-DAY RETURN AVERAGE



Source: Wikimedia Commons

return

The gain or loss obtained by taking the risk of investing.

Return is defined as the gain or loss obtained by taking the risk of investing. Return is usually measured over a specific period of time. The most common way to measure return is the holding period return (HPR). HPR is the total return, of an investment over a specific period. Holding period return is not an annualized return but instead it's the percentage return over a defined period. Return in investments is composed of a few components. Dividends, capital appreciation and interest received all comprise an investments return. Dividends are periodic payments made to investors from a company's profits, interest are period payments made for extending credit or lending money and capital appreciation is when there is an overall increase in an investments value over time. Expected return can be computed by listing possible outcomes and multiply them by the probability that particular outcome. The summation of the answer will result in the weighted average of the outcomes. A slight variation of the expected return formula can yield the expected value. Instead of using a list of returns, substitute an actual value (example 14). In a long position, holding period refers to the time between an asset's purchase and its sale. In a short sale, the holding period is the time between when a short seller initially borrows an asset from a brokerage and when he or she sells it back. In other words, the length of time for which the short position is held.

An investment's holding period is used for a number of different functions, including evaluating an investment's performance, calculating loss or gain from the investment and determining whether an investment is worthwhile. The holding period of an investment is also used to determine how the capital gain or loss should be taxed because long-term investments tend to be taxed at a lower rate than short-term investments.

Table 6.3 Measures of Return: Formula Method

Desired Value	Formula	Key
Investment return or holding period return (HPR)	$r_r = \frac{CF+EV-BV}{BV}$	r_r = rate of return CF = Cash Flow EV = ending value BV = beginning value
Arithmetic mean	$A_a = \frac{a^1+a^2}{2}$	
Geometric mean return or time-weighted return	$Ga = {}^{n}\sqrt{[(1+i)\times(1+i)\times\ldots(1+i)]}-1$	i = interest rate or rate of return n = number of periods
Expected return	$E(r) = \sum p_i \times r_i$	E(r) = expected return $\sum$ = summation p_i = probability of outcome i r_i = return of outcome i

Example 11: MoLamb stock was $60.80 per share at the end of last year. Since last year it paid $.65 per share in dividend. Currently the price is $76.50. If you owned 800 shares of MoLamb what was the dollar return and percentage return.

Answer:

Dollar Return = ($76.50 × 800) – ($60.80 × 800 + $0.65 × 800)

Dollar Return = $61,200 – 49,160 = $12,040.00

Percentage Return

$$r_r = \frac{CF + EV + BV}{BV}$$

$$r_r = \frac{\$.65 + \$76.50 - \$60.80}{\$60.80}$$

r_r = $16.35/$60.80 = 0.2689 or **26.89%**

In addition to investment return there are two other common measures of return. The two other common measures of return are the **arithmetic mean** and **geometric mean** return. The arithmetic mean is found by adding the rates together and dividing the number by the number of rates. This method does not take into account the effect of compounding. The geometric mean return is used for investments that involve compound interest. The geometric mean also must be utilized with percentages and it provides a more accurate measurement of what the actual average of the years studied.

Example 12: Calculate the arithmetic average of the following numbers: 20%, 10%, –25%, and –5%

Answer:

(20+10–25–5)/4 = **0%**

Example 13: An individual invests $1,000 in a money market account earn 20% in year 1 and 6% in year 2 and 1% in year 3. Calculate the geometric average return of the following numbers:

Answer:

$Ga = \sqrt[n]{[(1+i)\times(1+i)\times\ldots(1+i)]}-1$

$Ga = \sqrt[3]{[(1.20)\times(1.06)\times(1.01)]}-1$

$Ga = \sqrt[3]{1.2847} - 1$

$Ga = (1.2847)^{0.333} - 1 =$ **8.7%**

Example 14: MoLamb is considering a project with the following outcomes and their probabilities. What is the expected value E(v)?

Answer:

TABLE 6.4 EXPECTED VALUE OF A PROJECT

Assumptions	Probability	Outcome
Pessimistic	0.1	$4,500
Most Likely	0.6	$6,700
Optimistic	0.3	$9,345

$E(v) = \sum p_i \times v_i$

E(v) = [(0.10)(4,500)] + [(0.6)(6,700)]+[(0.3)(9,345)]

E(v) = [450] + [4,020]+[2,803.50]

E(v) = **$7,273.50**

6.6 Portfolio Efficiency

It is rare to invest in just one asset. However to calculate the expected rate of return of a single assets you take the summation of the probability of a return multiplied by the possible return. Probability is the likelihood that an event will occur by chance. The probability of a certain return occurring is usually based on certain economic conditions. Table 6.3 present an example of the computation required. Using the data in Table 6.3 compute the expected return and find it is 6.8%.

Example 15: MoLamb is considering a project with the following returns and their probabilities. What is the expected return E(r)?

TABLE 6.5 EXPECTED RETURN OF SINGLE ASSET

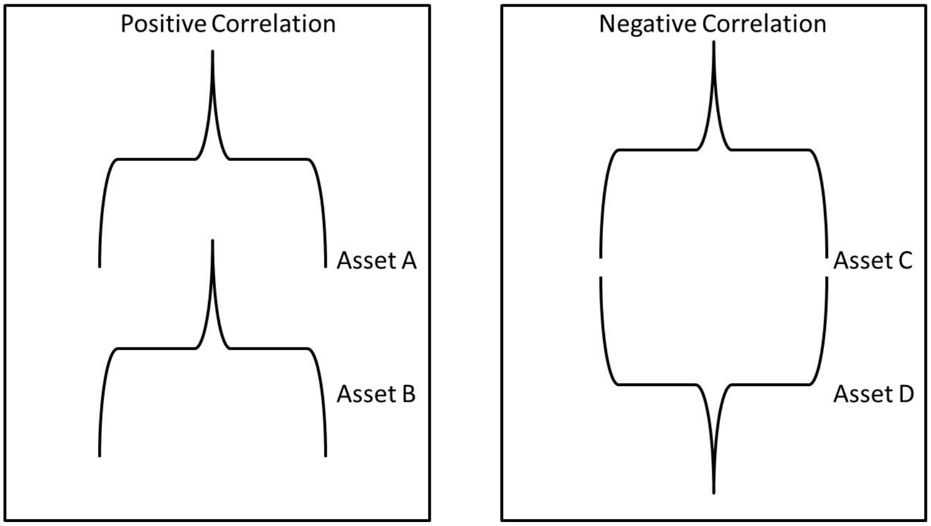
State of the Economy	Probability	Return
Fast Pace	0.1	50%
Slow Growth	0.6	8%
Recession	0.3	–10%

$E(r) = \sum p_i \times r_i$

E(r) = [(0.1)(.50)] + [(0.6)(.08)]+[(0.3)(–0.1)]

E(r) = 0.5 + 0.048 –0.03 = 0.086 or **6.8%**

FIGURE 6.4 PORTFOLIO CORRELATION



Portfolio. Instead of investing is a single asset, it is more likely to invest in a portfolio of assets. A portfolio is a group of financial assets. However, when choosing assets to make up the portfolio, they should be negatively correlated as depicted by Asset C and Assets D in Figure 6.4. Using the data in table 6.5 we will find that the expected return of the portfolio is 8.25%. The expected return of a portfolio is the weighted average return of each asset that makes up the portfolio.

Example 16: MoLamb is considering a project with the following returns and their probabilities. What is the expected portfolio return E(r) if you invest 20% in stock X, 50% in stock Y and 30% in stock Z?

TABLE 6.6 EXPECTED RETURN OF A PORTFOLIO

State of the Economy	Probability	Stock X	Stock Y	Stock Z
Expansion	0.20	18%	9%	6%
Normal	0.70	11%	7%	9%
Recession	0.10	–10%	4%	13%

$$E(Rp) = \sum_{j=1}^{m} w_j E(R_j)$$

E(r) Expansion= [(0.20)(18)] + [(0.5)(9)]+[(0.3)(6)] = 3.6 + 4.5 + 1.8= 9.9%

E(r) Normal = [(0.20)(11)] + [(0.5)(7)]+[(0.3)(9)] = 2.2 + 3.5 + 2.7 = 8.4%

E(r) Recession = [(0.20)(–10)] + [(0.5)(4)]+[(0.3)(13)] = –2.0 + 2.0 + 3.9 = 3.9%

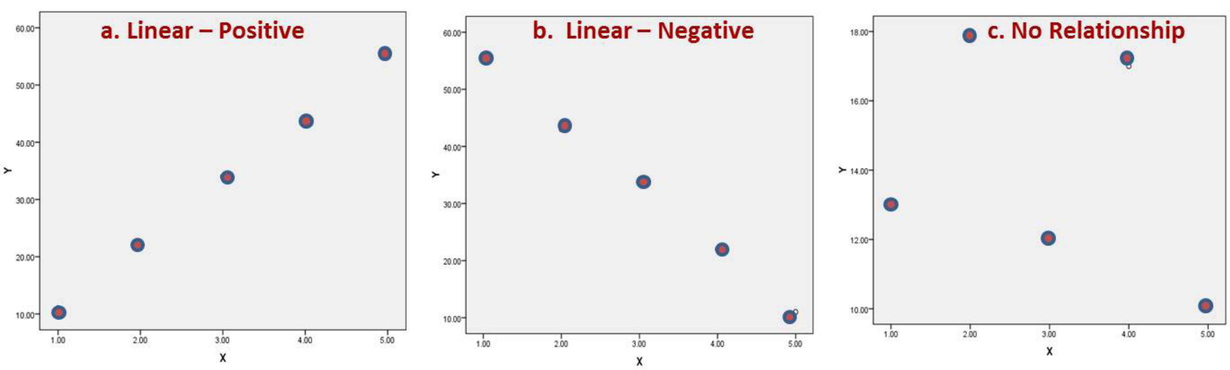
E(r) Portfolio = [(0.20)(9.9)] + [(0.7)(8.4)]+[(0.1)(3.9)] = 1.98 + 5.88+ 3.9 = 11.76%

A **correlation** is a statistical measure that is expressed as a number that describes the size and direction of a relationship between two or more variables. For example, let's say you want to find out if there is a relationship between "hours worked" and "income earned." There would be a relationship or correlation between the two if the increase in hours worked is associated with an increase in earned income. You can interpret correlation using a scatter plot, which can be subjective. A more precise way to measure the type and strength of a linear correlation between two variables is to calculate the correlation coefficient. The correlation coefficient is a measure of the strength and the direction of a linear relationship between two variables. The letter *r* represents the sample correlation coefficient. The formula for *r* is given below.

$$r = \frac{\sum_{i}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i}^{n}(x_i - \bar{x})^2 \sum_{i}^{n}(x_i - \bar{x})^2}}$$

The formal name for *r* is the Pearson product moment correlation coefficient. It is named after the English statistician Karl Pearson. The range of the correlation coefficient is –1 to 1. If x and y have a strong positive linear correlation, *r* is close to 1; the relationship would be closely related to scatter plot (a.) in Figure 6.5. If x and y have a strong negative linear correlation, *r* is close to –1; the relationship would be closely related to scatter plot (b.) in Figure 6.5. If there is no linear correlation or a weak linear correlation, *r* is close to 0; the relationship would be closely related to scatter plot (c.) in Figure 6.5. The independent explanatory variable X is measured by the horizontal axis and the dependent, response variable Y is measured by the vertical axis. The fact that 2 variables are strongly correlated does not imply a cause-and-effect relationship between the variables. More in-depth study is usually needed to determine whether there is a so called causal relationship between the variables. Most stock returns have a positive correlation with each other over time and have a correlation coefficient close to .65.

FIGURE 6.5 SCATTER PLOT



6.7 Unsystematic Risk (Diversifiable)

The following are a few examples of unsystematic risk: Company risk, default risk, financial risk, credit risk, event risk and political risk. Company risk is risk related to a particular business or company. If Company X went on strike or went bankrupt, it would only affect company X. Financial risk is concerned with the degree or leverage or debt a firm is utilizing. A highly leverage firm is considered to be more risky. Default risk is closely associated with financial risk. Highly leverage firms are more prone to default that other less levered firms. Credit risk is related to the chance of a firm at defaulting. The credit rating of a firm is related to the risk associated with a firms default risk. Event risk is associated with the possibility of an unanticipated damaging event occurring. Political risk is the type of risk that occurs to firms who operate in a threatening political climate. A political climate is more risky; it is unstable, has a poor economic system or operates under a state sponsored regime.

6.8 Systematic Risk (Non-diversifiable)

The following are a few examples of systematic risk which can be remembered using the word PRIME: purchasing power risk (inflation), reinvestment risk, interest rate risk, market risk and exchange rate risk. When interest rates increase overall, it affects many types of investments. However, bonds in particular are more affected by the interest rate shift. When inflation rises, purchasing power will decrease. It will take more dollars to buy the same basket of goods. Inflation is a macroeconomic variable which can affect the entire market. If investors feel that the market is overvalued or fear new legislation and its effect on invested assets, this can cause the overall market to decrease in value.

6.9 Beta Coefficient

The **beta (β) coefficient** is a measure for systematic risk. Since diversification does not reduce systematic risk, there must be a method to account for this type of risk. Beta signifies the degree to which a single asset or portfolio is affected by market factor. Beta is represented by the slope of the **Security Market Line (SML)**, Figure 6.2. The Beta for the market (for example S&P 500) is equal to 1. If an asset or portfolio is grater that 1 it is consider to me more risky than the market. If an asset or portfolio is less than 1 it is considered less risky than the market. See table 6.8 for select beta values.

beta coefficient

A measure for systematic risk.

Security Market Line (SML)

Depicts the relationship between risk (as measured by beta) and return.

TABLE 6.7 SELECT BETA VALUE'S

Company Name	Symbol	Google Finance	Yahoo Finance
Apple	AAPL	1.04	0.74
Home Depot	HD	0.86	0.99
Microsoft	MSFT	0.93	1.14
Caterpillar	CAT	1.92	1.84
J. C. Penny	JCP	1.86	1.98
Bank of America	BAC	2.38	1.76
General Electric	GE	1.62	1.47
Texas Instruments	TXN	1.11	1.15
Primerica	PRI	----	0.62
Wal-Mart	WMT	0.35	0.39

Source http://finance.yahoo.com and www.google.com/finance March 2013

6.9a Calculating Beta

The beta is change in percentage return on a security divided by the change in percentage return on the market. Beta can be computed using simple linear regression. Any point on the regression line, also called a line of best fit (or least squares lines), represents the mean for each x-value. In algebra, you learn that you can write an equation of a line by finding it slope letter M and Y intercept B. The equation has the form Y=Mx+B. Similarly, the equation of a regression line (or regression model) allows you to use the independent variable X to make predictions for the dependent variable Y.

Simple linear regression assumes the relationship between the y-variable (dependent or response variable) and the x-variable (independent or predictor variable) are approximated by a straight line; thus denoting its linearity. Initial inspections of the scatter plot for the y versus x variables, as long as the (x, y) points fall generally along the least squares line, then we can assume a linear relationship. Similar to the equation for a line, in simple linear regression, we are able to create a model by which we can make predictions based on the given data. NOTE: Extrapolation outside the boundaries (smallest and largest x, y points) of the least squares line is forbidden. The regression model used to make predictions of y (or the response variable), given an x (predictor variable) is defined as:

$$y = \beta_0 + \beta_1 x + \varepsilon$$

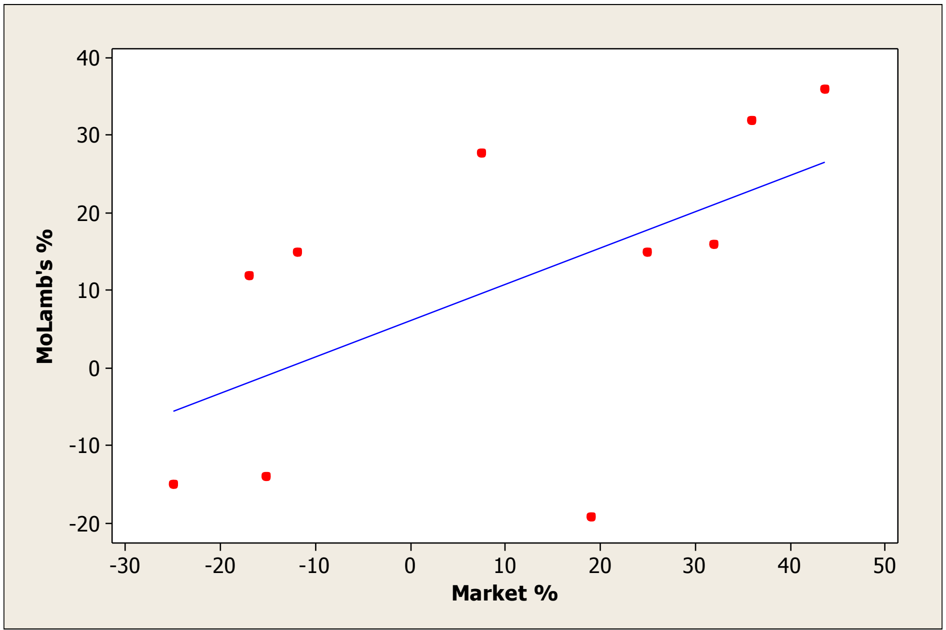
Where y is the response based on the summation of β_0 (intercept) and the product of $\beta_1 x$ (slope*predictor value). Epsilon (ε) represents the variation of the least squares line and are in fact the residuals (or deviations) from the least squares line. The sum of ε are assumed to equal zero.

Table 6.8 Returns for MoLamb and Market

Year	Stock: MoLamb%	Market%
1	36	43.6
2	15	–12
3	–14	–15.3
4	16	32
5	27.8	7.5
6	15	25
7	32	36
8	12	–17
9	–15	–25
10	–19.3	19

Using the information in table 6.9 the beta was computed using Minitab Statistical Software but can also be computed using a financial calculator. The resulting beta (which equals β_1 or the slope of the least squares line) was 0.470. The individual stock is the dependent (Y) variable and the market is the independent (X) variable. See the following output from Minitab statistical software:

Figure 6.6 Scatter Plot of MoLamb's % versus the Market %



Regression Analysis: MoLambs % versus Market %
The regression equation is

MoLambs % = 6.14 + 0.470 Market %

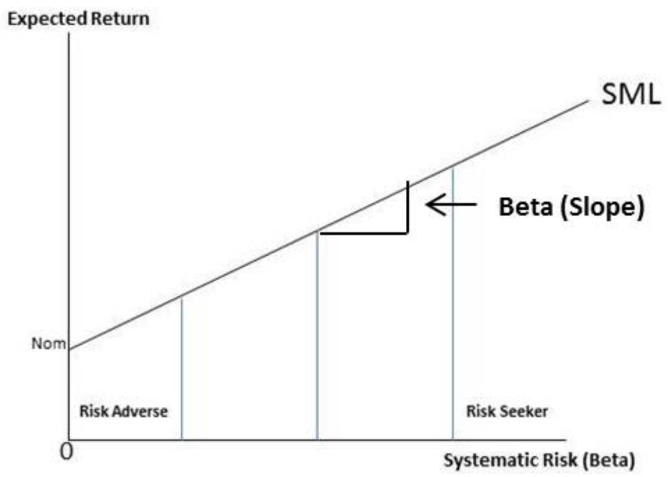
Predictor	Coef	SE Coef	T	P
Constant	6.144	5.851	1.05	0.324
Market %	0.4697	0.2285	2.06	0.074

The graph shows a positive linear relationship, although through additional statistical testing the relationship is medium to medium/low. Within the regression analysis, the regression model (regression equation) is shown where MoLamb's % is the y-variable (or the response variable) and Market % is the x-variable (or the predictor variable). The intercept for the least squares line is given as 6.144 and the slope of the line is 0.470 (which is the Beta we are looking for). In order to make a prediction of MoLamb's %, find the product of Market % and 0.470, then add to 6.144.

6.10 Security Market Line

The security market line (SML) depicts the relationship between risk (as measured by beta) and return. As perceived risk or uncertainty rises, the required return also rises. Investors usually follow their risk tolerance level when choosing assets. The SML gives a good indication of the expected return based on risk tolerance level. As an investors risk tolerance changes, there will be movement along the SML. The slope of the SML is depicts the return necessary per unit of risk.

FIGURE 6.7 SECURITY MARKET LINE



6.11 Portfolio Theory and Capital Asset Pricing Model (CAPM)

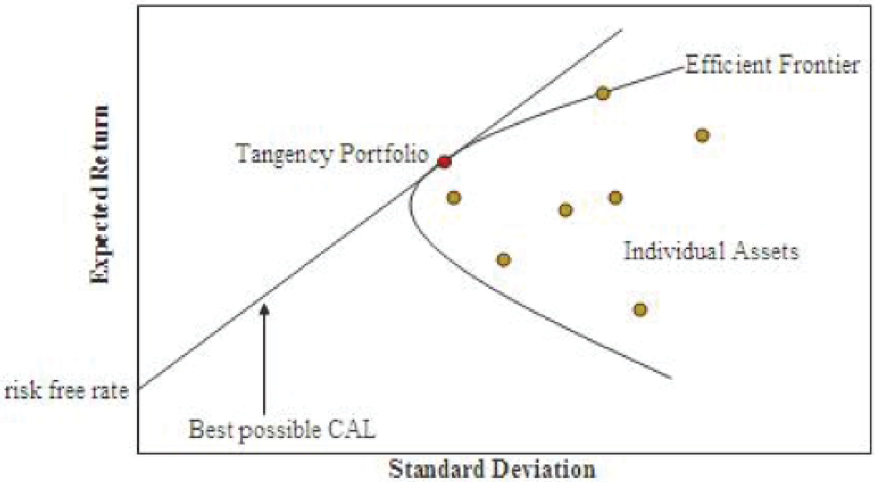
6.11a Modern Portfolio Theory

Modern portfolio theory is a finance theory that was introduced by Harry Markowitz in 1952, that attempts to achieve the highest expected return of a portfolio. Markowitz showed that the variance of a portfolio's rate of returns was an important measure of a portfolios' risk. Markowitz showed the important of portfolio construction as a method to reduce overall risk. Markowitz portfolio theory also led to the development of the concept of a risk-free asset which was necessary for construction of the Capital Asset Pricing Model (CAPM). According to Markowitz, for every point on the efficient frontier, there is at least one portfolio that can be constructed from all available investments that has the expected risk and return corresponding to that point. The efficient frontier is curved because there is a diminishing marginal return to risk. Each unit of risk added to a portfolio gains a smaller and smaller amount of return.

Modern Portfolio theory

A finance theory introduced by Harry Markowitz in 1952 that attempts to achieve the highest expected return of a portfolio.

FIGURE 6.8 EFFICIENT FRONTIER



6.11b Capital Asset Pricing Model (CAPM)

The capital asset market pricing model assists investor in determining the required rate of return by accounting for systematic risk. By using beta as the risk measures allows the investor or financial manager to find the risk premium necessary to make an investment decision. The CAPM is depicted graphically in the SML in figure 6.2 and the formula for CAPM is depicted in table 6.10. To use the Capital Asset Pricing Model (CAPM) there are certain assumptions that must be made.

Capital Asset Pricing Model (CAPM)

Assists investors in determining the required rate of return by accounting for systematic risk.

6.11c CAPM Assumptions

- The market is competitive and there are many investors competing in the market.
- There is no restriction on access to securities.
- Investors have the same investment time horizon.
- All investors can borrow or lend at the risk free rate.
- Investors are holding a well diversified portfolio
- Investors must be compensated for market risk (systematic risk)

TABLE 6.9 CAPITAL ASSET PRICING MODEL: FORMULA METHOD

Desired Value	Formula	Key
Required rate of return (CAPM)	$r_r = R_f + \beta(r_m - R_f)$	r_r = rate of return R_f = risk free rate (t-bill) β = beta (systematic risk) r_m = market return $(r_m - R_f)$ = market risk premium

Example 17: If the risk-free rate of return is 15%, beta is 1.50, and the market risk premium is 5.6% what is the required rate of return?

Answer:

$r_r = R_f + \beta(r_m - R_f)$

$r_r = 15 + 1.50(5.6)$

$r_r = 23.4\%$

Example 18: If the market has a required rate of return of 12.6%, corporate bond yields 16.9%, and 20-year government bond yields 9.5% and a U.S. Treasury bill yields 4.5%. What is the market risk premium?

Answer:

Risk premium = 12.6% – 4.5% = 8.1%

Example 19: If the risk-free rate of return is 15%, beta is 1.50, and the market risk premium is 5.6% what is the required rate of return?

Example 20: Determine the value of beta for an asset with a required rate of return of 17.4%, given a risk-free rate of 4% and a market return of 12.4%.

Answer:

$r_r = R_f + \beta(r_m - R_f)$

$17.4= 4 + \beta(12.4 - 4)$

$17.4=4+8.4\beta$

$13.4 =8.4\beta$

$\beta = 1.595$ or **1.6**

Example 21: Determine the risk-free rate for an asset, given a required rate of return of 14.5%, a beta of 0.77 and market return of 18%.

Answer:

$r_r = R_f + \beta(r_m - R_f)$

$14.5 = R_f + 0.77(18 - R_f)$

$14.5 = R_f + 13.86 - 0.77R_f$

$14.5 = 0.23R_f + 13.86$

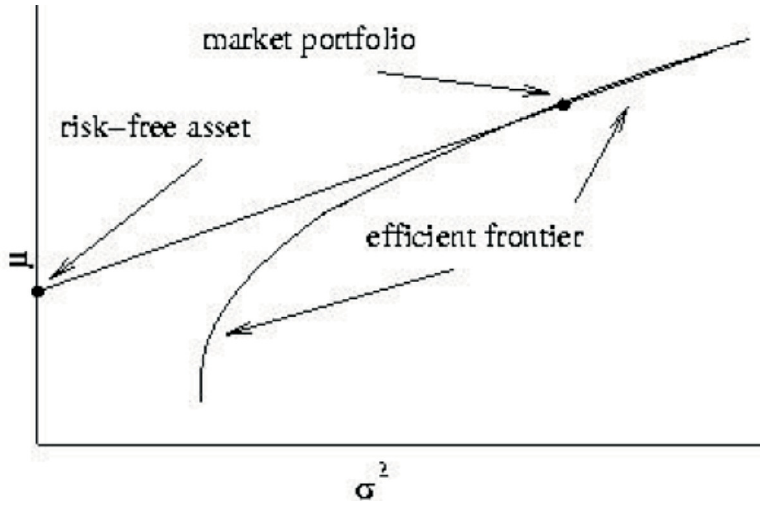
$0.64 = 0.23R_f$

$R_f = \mathbf{2.78}$

6.11d Capital Market Line

The capital market line is a line used in the capital asset pricing model to illustrate the rates of return for efficient portfolios depending on the risk-free rate of return and the level of risk (standard deviation) for a particular portfolio.

FIGURE 6.9 CAPITAL MARKET LINE



KEY POINT: Stock Market Equilibrium

The stock market is in equilibrium when the expected return on the stock is equal to the required return on the stock (as given by the CAPM model). If, for example, a company announces good news its expected return increases above its required return. Investors will buy the stock, pushing up its price and reducing its expected return to again equal its required return. The stock price is then back in equilibrium until the next positive or negative news event. Does this occur quickly? Yes, the stock market is very efficient at digesting good news or bad news into the stock price. Financial analysts scour our stock market in search of underpriced (or overpriced) stocks, and it does not take long for them to discover stock prices in disequilibrium and make money from it.

6.12 Risk Adjusted Return Measurements

Investors evaluating fund or manager performance often rely on risk-adjusted performance measures, such as the information ratio, the Sharpe ratio, Treynor and alpha, to help them choose appropriate investments as well as to understand past performance.

6.12a Information Ratio

The information ratio (IR) is a ratio of portfolio returns above the returns of a benchmark (usually an index) to the volatility of those returns. The information ratio (IR) measures a portfolio manager's ability to generate excess returns relative to a benchmark, but also attempts to identify the consistency of the investor. This ratio will identify if a manager has beaten the benchmark by a lot in a few months or a little every month. The higher the IR the more consistent a manager is and consistency is an ideal trait.

Formula: (Portfolio Return – Return of Index or Benchmark)/ Tracking error

Tracking Error: Standard deviation of the active return.

6.12b Sharpe Index

The Sharpe index (sometimes called the Sharpe ratio) relates the return on an investment or portfolio to the degree of total risk taken. The higher the index quotient, the greater is the return for each unit of risk. By itself, the Sharpe index is of little use, but when used in comparing other mutual funds it is very useful in determining which fund provided the most return per unit of risk. The fund with the highest Sharpe ratio would be chosen.

Formula: (Portfolio Return – Risk-Free Rate) / Standard Deviation

6.12c Treynor Index

The Treynor and the Sharpe indices are closely related to each other. While the Sharpe index relates return to total risk, the Treynor index relates the return of an investment or portfolio to the degree of systematic risk taken. As with the Sharpe index, the higher the index quotient, the greater is the return for each unit of risk. This index is useful when compared to other diversified funds. The risk measure is beta; therefore the index should be used to evaluate the performance of diversified portfolios only (because of this qualification, it is used less often than the Sharpe index).

Formula: (Portfolio Return – Risk-Free Rate) / Beta

6.12d Jensen Index (Alpha)

A financial model, called the capital asset pricing model (CAPM), is one commonly accepted model to determine the rate of return an investor should expect from an investment for a given level of systematic risk, as measured by beta. The Jensen index carries this concept one step further. It compares the expected return with the actual return. The difference is called the Jensen index or, more commonly, called alpha. It indicates the additional return, if any, earned by the portfolio manager after adjusting for risk, as measured by beta. A high positive alpha is considered a desirable feature in an investment; it means that the investment actually produced a greater return than its risk level

Formula: Jensen\'s Alpha = Portfolio Return – Benchmark Portfolio Return

Chapter 6 Summary

1. **What are four common measures of dispersion?** Four common measures of dispersion are the range, variance, standard deviation and coefficient of variation. The simplest measure of variability or dispersion is the **range.** The **variance** is the average of the squared differences from the mean. It is a measure of how far each number or value in the data set is from the mean. The **standard deviation** is a measure of how spreads out the numbers are from the mean. The **coefficient of variation** is a relative measure of the standard deviations size in relationship to the mean. The standard deviation is an absolute number and addresses the risk side. The coefficient of variation is necessary to assess the actual amount of variation represented by the standard deviation.

2. **What is the Security Market Line (SML) and what does it measure?** The security market line (SML) depicts the relationship between risk and return. As perceives risk or uncertainty rises, the required return also rises. Investors usually follow their risk tolerance level when choosing assets. The SML gives a good indication of the expected return based on risk tolerance level.

3. **What is the capital asset pricing model and what is it used for? Capital Asset Pricing Model (CAPM).** The capital asset market pricing model assists investor in determining the required rate of return by accounting for systematic risk. By using beta as a risk measures allows the investor of financial manager to find the risk premium necessary to make an investment decision.

Chapter 6 Key Words

Term	Page
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systematic/non-diversifiable risk	104
unsystematic/diversifiable risk	104

Chapter 6 Problems

PR-1 Calculate the arithmetic mean and geometric mean return of the following rates of return: 4%,15%, 9% and 6%.

PR-2 Compute the sample standard deviation for the following numbers: 19,18,3,5,6,–9,18,–12,22

PR-3 Using the following table of MoLamb Stock

Year	Beginning Stock Price	Ending Stock Price
2010	$12.56	$19.35
2011	$23.00	$56.00
2012	$67.30	$37.50
2013	$89.65	$121.58
2014	$35.67	$14.58

A. Compute the rate of return for each year for MoLamb Stock.
B. Calculate the sample standard deviation and coefficient of variation of the returns.

PR-4 Using the table the following table:

Rate of Return	Probability
6%	0.10
8%	0.10
12%	0.25
21%	0.15
35%	0.15
40%	0.15
45%	0.10

A. Compute the range of possible rates of returns
B. Compute the standard deviation.
C. Compute the coefficient of variation of the returns.

PR-5 Calculate the required return for an asset, with a risk-free rate of 8%, market return of 14%.
A. That has a beta of 1.3.
B. That has a beta of 0.9
C. That has a beta of 2

PR-6 Determine the value of beta for an asset with a required rate of return of 18%, given a risk-free rate of 7% and a market return of 13%.

PR-7 Determine the risk-free rate for an asset, given a required rate of return of 19%, a beta of 1.35 and market return of 16%.

PR-8 Using the following data, calculate the beta coefficient using linear regression.

Returns for MoLamb and Market

Year	Stock: MoLamb%	Market%
1	26	43.6
2	12	–12
3	–14	–15.3
4	16	32
5	19.8	7.5
6	15	25
7	32	15
8	12	–17
9	–15	–25
10	–15.5	19

PR-9 MoLamb Corp stock was $60.00 per share at the end of last year. Since then, it paid a $1.00 per share dividend last year. The stock price is currently $62.50. If you owned 400 shares of Molamb, what was your percent return?

PR-10 MoLamb stock was $28 per share at the end of last year. Since then, it paid a $1.55 per share dividend last year. The stock price is currently $23. If you owned 1300 shares of MoLamb, what was your percent return?

PR-11 An investor owns $2,500 of MoLamb stock, $4,000 of Dirty Oil Inc., and $6,000 of Hole in the Wall Furniture. What are the portfolio weights of each stock?

PR-12 Year-to-date, Company Z had earned a –2.10 percent return. During the same time period, Company Q earned 8.00 percent and Company X earned 6.25 percent. If you have a portfolio made up of 40 percent Company Z, 30 percent Company Q, and 30 percent Company X, what is your portfolio return?

PR-13 The past five monthly returns for MoLamb are 6.25 percent, 8.13 percent, –2.05 percent, 4.25 percent, and 9.25 percent. What is the average monthly return?

PR-14 The standard deviation of the past five monthly returns for MoLamb are 4.25 percent, 2.13 percent, –2.05 percent, 9.25 percent, and 6.75 percent. What is the average monthly return?

PR-15 The standard deviation of the past five monthly returns for K and Company are 2.28 percent, 2.64 percent, –1.05 percent, 4.25 percent, and 9.25 percent. What is the average monthly return?

PR-16 At the beginning of the month, you owned $6,000 of Company G, $8,000 of Company S, and $1,000 of Company N. The monthly returns for Company G, Company S, and Company N were 7.25 percent, –1.50 percent, and –0.23 percent. What is your portfolio return?

PR-17 The past five monthly returns for PG Company are 1.25 percent, –1.50 percent, 4.25 percent, 3.75 percent, and 1.98 percent. What is the average monthly return?

PR-18 If you own 1000 shares of Alaska Corporation at $19.95, 250 shares of Best Company at $17.50, and 250 shares of Motor Company at $2.50, what are the portfolio weights of each stock?

PR-19 At the beginning of the month, you owned $15,500 of No Money Records, $4,500 of Hammer Toes, and $9,000 of Shiver Lips. The monthly returns for No Money Records, Hammer Toes, and Shiver Lips were 7.10 percent, –1.36 percent, and –0.54 percent. What is your portfolio return?

PR-20 If you invested $1,000 in MoLamb and $5,000 in Bad Time and the two companies returned 15% and 18% respectively, what was your portfolio's return?

PR-21 You have a $1,000 portfolio which is invested in stocks A and B plus a risk-free asset. $400 is invested in stock A. Stock A has a beta of 1.5 and stock B has a beta of .75. How much needs to be invested in stock B if you want a portfolio beta of .90?

PR-22 You recently purchased a stock that is expected to earn 12% in a booming economy, 8% in a normal economy and lose 5% in a recessionary economy. There is a 15% probability of a boom, a 75% chance of a normal economy, and a 10% chance of a recession. What is your expected rate of return on this stock?

PR-23 The rate of return on the common stock of Flowers by Flo is expected to be 14% in a boom economy, 8% in a normal economy, and only 2% in a recessionary economy. The probabilities of these economic states are 20% for a boom, 70% for a normal economy, and 10% for a recession. What is the variance of the returns on the common stock of Flowers by Flo?

PR-24 You own a portfolio with the following expected returns given the various states of the economy. What is the overall portfolio expected return?

Economic State	Probability	Rate of Return
Expansion	15%	19%
Normal	50%	12%
Recession	35%	–12%

PR-25 What is the variance of a portfolio consisting of $3,700 in stock P and $7,500 in stock Q?

Economic State	Probability	Rate of Return Stock P	Rate of Return Stock Q
Expansion	25%	19%	8%
Normal	70%	7%	5%

PR-26 What is the beta of a portfolio comprised of the following securities?

Security (Stock)	Invested Amount	Beta
X	$5,600	1.04
Y	$6,700	1.35
Z	$9,789	.95

PR-27 You would like to combine a risky stock with a beta of 1.5 with U.S. Treasury bills in such a way that the risk level of the portfolio is equivalent to the risk level of the overall market. What percentage of the portfolio should be invested in Treasury bills?

PR-28 The risk-free rate of return is 4% and the market risk premium is 8%. What is the expected rate of return on a stock with a beta of 1.28?