What Is Algebra?

Where did algebra come from?

There are about 100,000 different stories regarding algebra. The origin of algebra dates back to the ancient Babylonians. They developed mathematical systems that enabled them to calculate by applying their own formulas rather than utilizing geometric methods as most Egyptians, Greek, Indian, and Chinese mathematicians of that era did to solve problems.

The word "algebra" is of Arabic origin. It comes from the word *al-jar*, meaning reunion. Al-jar comes from the title of the book *ilm al-jebr wa'l-muqabala* written in 820 by the Persian mathematician Mahommed ben Musa al-Khwarizmi. The English translation of the title is *The book of Summary Concerning Calculating by Transposition and Reduction.*

Studying algebra will is very useful because it allows the general formulation of arithmetical laws, equations, and functional relationships. Generally, algebra is the study of the properties of operations on real numbers. Before we use the general formulation of arithmetical laws, we will learn about the properties of the real number system.

You may think that algebra is so difficult it will cause you headaches. Algebra is not easy; but once you learn it, you will find it useful in everyday situations. For example, if you have 5 apples in your refrigerator and you need 8 apples to make a pie, how many more apples do you need to purchase at the store? With such simple numbers, you easily come up with the answer 3 apples. However, in algebra we would use an equation to solve for an unknown number—in this case the number of additional apples you need. We write the unknown number as x. So I can write

$$5 + x = 8$$

We call x the variable which represents the unknown number of apples. Solving this equation means finding the unknown number x.

By substituting $x = 3$, we have

$$5 + 3 = 8 \text{ apples}$$

Isn't it fun to find the unknown number? Math is like magic.

Theses are the kinds of things that we will learn throughout this textbook. Let's start learning some fantastic mathematical magic!

To the Instructor

The authors are indebted to the faculty at those institutions that have adopted and have steadfastly continued to use the earlier editions of this text. Your letters and comments are genuinely appreciated and will always receive a prompt response.

Objectives of This Text

This second edition of *Beginning Algebra with Critical Thinking* maintains our objective of providing a textbook designed for *use by the student.* We have adopted an informal, supportive style to encourage the student to read the book and to develop confidence under its guidance. We introduce concepts first by example with accompanying diagrams and illustrations that bolster the "reasonableness" of the resulting rules. We immediately reinforce every new mathematical technique or result with fully worked-out examples and captions clarifying their purpose. After each example, the student is presented with the opportunity to tackle a parallel problem, called a **Progress Check**, with the answer following it in the text.

New in This Edition

Although the structure of earlier editions has been retained, we have made significant additions and improvements in this edition.

- Completely new chapter openers with motivational applications and references to many websites of mathematical interest.
- Brand new chapter projects at the end of each chapter.
- Many new exercises, most of which emphasize the use of graphing calculators.
- New explanatory material for graphing calculator use.

Pedagogic Devices

We have continued to employ those pedagogic devices that instructors have found useful in the earlier editions.

Split Screens

Many algebraic procedures are described with the aid of a "split screen" that displays simultaneously both the steps of an algorithm and a worked-out example.

✔ Progress Checks
A problem (with answers) accompanies every numbered example in the text to enable the student to test his or her understanding of the material just described.

Warnings
To help eliminate misconceptions and prevent bad mathematics habits, we have inserted numerous **Warnings** that point out the incorrect practices most commonly found in homework and exam papers.

Vignettes

In each chapter we have inserted one or more vignettes, elements that are independent of the text yet are often related to the mathematical concepts. The vignettes are intended to catch the attention of the student and heighten interest in the material. (We hope they will provide interesting reading for the instructor as well.)


Exercises

Abundant, carefully graded exercises provide practice in the mechanical and conceptual aspects of algebra. Exercises requiring the use of a calculator or graphing calculator are indicated by the calculator icons shown to the left. Exercises of a more challenging nature are indicated by a *. Answers to odd-numbered exercises, review exercises, and progress tests appear in an appendix at the back of the book.

End-of-Chapter Material

Every chapter contains a summary that includes the following:

Terms and Symbols with appropriate page references

Key Ideas for Review to stress the concepts

Review Exercises to provide additional practice

Progress Tests to provide self-evaluation and reinforcement

Chapter Projects

This edition has added chapter openers and related projects at the end of each chapter. These emphasize additional applications and demonstrate the widening relevance of

algebra in many areas. Some feature a "look ahead" to topics of future courses, like polynomial curve fitting or Calculus applications. Students may be excited by the career possibilities suggested by some of these sections. Instructors may wish to review the projects first; they provide one possible means of selecting which material and which exercises to emphasize. Some of the projects include essay components; the increasing importance of communication skills in scientific careers and upper-level courses seems to make this new emphasis advisable. Projects could be modified or expanded to involve students working in groups. Significant opportunities for Internet research and graphing calculator exploration are also offered.

Answers

The answers to all **Review Exercises** and **Progress Tests** appear in the back of the book.

Solutions

Worked-out solutions to selected **Review Exercises** appear in a separate section at the back of the book. The solved problems provide one more level of reassurance to the student using the **Review Exercises** in preparation for the **Progress Tests**.

A Note on the Use of Calculators

Some of the new exercises in this edition call for the use of a graphing calculator. It is recommended that all students have one, and read the manual in order to become proficient in its use. Many instructors view graphing calculators as essential tools for students of algebra, and their use becomes even more pronounced in more advanced courses. The aid they provide in visualization, rapid evaluation of functions, using graphs to discover unsuspected relationships between concepts, and offering new approaches and a greater variety of problem-solving methods, more than balances the additional challenge in becoming skilled and comfortable with them.

Many models of graphing calculator are now available, and most offer the additional benefit of links which can be used to share programs and applications between students, from instructor to student, or from the Internet.

Supplementary Material

Student Solutions Manual by Jorge Cossio

Instructor's Manual with Tests by Gail Edinger

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To the Student

This book was written for you. It gives you every possible chance to succeed—if you use it properly.

We would like to have you think of mathematics as a challenging game—but not as a spectator sport. This wish leads to our primary rule: *Read this textbook with pencil and paper handy.* We illustrate every new idea or technique with fully worked-out examples. As you read the text, carefully follow the examples and then do the **Progress Checks**. The key to success in a math course is working problems, and the **Progress Checks** are there to provide immediate practice with the material you have just learned.

Your instructor will assign homework from the extensive selection of exercises that follows each section in the book. *Do the assignments regularly, thoroughly, and independently.* By doing many problems, you will develop the necessary skills in algebra, and your confidence will grow. Since algebraic techniques and concepts build on previous results, you can't afford to skip any of the work.

To help prevent or eliminate improper habits and to help you avoid the errors that we see each semester as we grade papers, we have interspersed **Warnings** throughout the book. The **Warnings** point out common errors and emphasize the proper method. They are summarized at the end of the chapter under the heading **Common Errors**.

We provide important review material at the end of each chapter. The **Terms and Symbols** should all be familiar by the time you reach them. If your understanding of a term or symbol is hazy, go back and read the definition.

It is possible to become so involved with the details of techniques that you lose track of the broader concepts. The list of **Key Ideas for Review** at the end of each chapter will help you focus on the principal ideas.

The **Review Exercises** at the end of each chapter can be used as part of your preparation for examinations. The section covering each exercise is indicated so that, if needed, you can go back to restudy the material. If you get stuck on a problem, see if the problem that is giving you difficulty or a similar problem is numbered in bold, indicating that a worked-out solution appears in the back of the book in the Solutions appendix. You are then ready to try **Progress Test A**. You will soon pinpoint your weak spots and can go back for further review and more exercises in those areas. Only then should you proceed to **Progress Test B**.

We believe that the eventual "payoff" in studying mathematics is an improved ability to tackle practical problems in your field of interest. To that end, this book places special emphasis on word problems, which recent surveys show often trouble students. Since algebra is the basic language of the mathematical techniques used in virtually all fields, the mastery of algebra is well worth your effort.