CHAPTER 1

Real Numbers and an Introduction to Algebra

Objectives

In this chapter you will learn how to do the following:

1. Read mathematical symbols.
2. Distinguish between different sets of numbers.
3. Add, subtract, multiply, and divide signed numbers.
4. Apply the **order of operations**.
5. Evaluate expressions.

Introduction Problem

While you may not know how to solve the problem below now, you should be able to solve this introduction problem by the end of the chapter.

The total number of hours worked per day by all the registered nurses in California can be modeled by the equation $n = \frac{22}{3}t + 100$, where n is the number of hours worked per day in units of thousands and t is the number of years after 1976. Use the model to complete the table below:

Year	t	n
2006	30	$n = \frac{22}{3}(30) + 100 \quad 320$
2012	36	$n = \frac{22}{3}(36) + 100 \quad 364$
2015		

(Health Service Research)

1.1 Symbols and Sets of Numbers

Symbols

1. The statement $-10 < -5$ is read, "Negative 10 is less than negative 5."
   Note: You can also read the statement $-10 < -5$ from right to left as, "-5 is greater than -10."

2. The statement $-12 > -15$ is read, "Negative 12 is greater than negative 15."
   Note: You can also read the statement $-12 > -15$ from right to left as, "-15 is less than -12."

3. The statement $x \leq -5$ is read, "x is less than or equal to negative 5."
   Note: The statement $-5 \geq x$ read from right to left is, "x is less than or equal to negative 5."

4. The statement $x \geq -5$ is read, "x is greater than or equal to negative 5."
   Note: The statement $-5 \leq x$ read from right to left is, "x is greater than or equal to negative 5."

5. The statement $|-5| = 5$ is read, "The absolute value of negative 5 equals 5."
   Note: The absolute value of a number gives you the distance the number is from zero. Therefore, the absolute value of a positive or a negative number will be a positive number.

Sets of Numbers

The numbers {1,2,3 …} are called counting numbers.
This set of numbers is also known as natural numbers or positive integers.

Example 1: Populations are counting numbers as long as the species is not extinct. If they are extinct, then the population is 0. 0 is not a natural number. Populations cannot be negative or fractional.

The numbers {0,1,2,3 …} are called whole numbers.

Example 2: Populations are whole numbers even if they do become extinct. Populations cannot be negative or fractional.

The numbers {… –3,–2,–1,0,1,2,3 …} are called integers.

Example 3: The outside temperature would be an integer, if it is given to the nearest degree. In many places, it can be negative as well as positive.

Any number that can be expressed as a fraction $\frac{a}{b}$ where both a and b are integers and b does not equal 0 is a rational number.

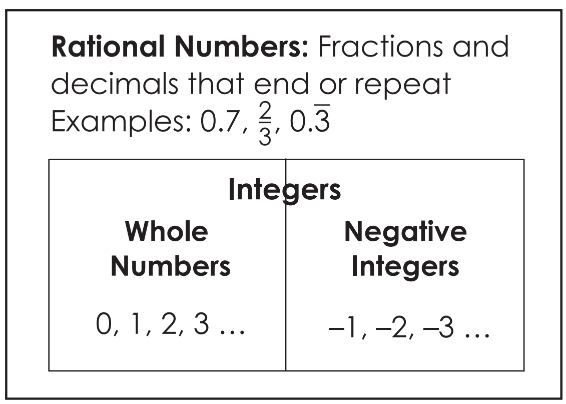
Notes:

1. Any integer is a rational number. For example, 4 can be expressed as $\frac{4}{1}$.
2. Any number expressed as a terminating decimal (a decimal that ends) or repeating decimal is a rational number. For example, 0.33 can be expressed as $\frac{33}{100}$.

Example 4: The elevation (The elevation could be positive, 0, negative, or a fraction.)

Example 5: Temperatures (The temperature could be positive, 0, negative, or a fraction.)

FIGURE 1.1 RATIONAL NUMBERS



1.1 Exercise Set

To what set(s) do the following numbers belong?

1) $\frac{1}{2}$

2) $1\frac{3}{4}$

3) –5

4) 0

5) 0.3

6) $0.\overline{3}$

Evaluate the following.

7) $|-10|$

8) $-|-10|$

9) $|10|$

10a) Which number is farther from zero, –2 or –5?

10b) Which number is farther from zero, –7 or 5?

10c) Which number is farther from zero, 7 or –5?

Are the following true or false?

11) $5 < 7$

12) $5 \leq 7$

13) $-5 < -7$

14) $-5 \geq -7$

15) $|-10| > |5|$

1.2 Addition and Subtraction of Signed Numbers

When adding or subtracting signed numbers, think of temperatures or money or height above ground. For example, if the problem is –7 – 3, think of the temperature being –7, then it goes down 3 degrees. What temperature would it be?

Working with Positive and Negative Decimals and Fractions

1) Which temperature is colder, 20 degrees or 10 degrees?

2) How much colder is 10 degrees compared to 20 degrees?

   Solution: If you want to find the difference between two numbers, then you will need to subtract the 10 from 20. So, 20 – 10 gives us 10.

3) Which temperature is colder, –20 degrees or –10 degrees?

   Solution: –20 is colder because it is farther below zero.

4) Which temperature is colder, $\frac{1}{4}$ of a degree or $\frac{1}{2}$ of a degree?

   Solution: $\frac{1}{4}$ is colder because it is smaller than $\frac{1}{2}$.

5) Which temperature is colder, $\frac{-1}{4}$ of a degree or $\frac{-1}{2}$ of a degree?

   Solution: $\frac{-1}{2}$ is colder because it is farther below zero than $\frac{-1}{4}$.

6) If the temperature at 8:00 a.m. is –20 degrees and the temperature at noon is 10 degrees, how much did the temperature increase?

   Solution: You will need to subtract these two numbers: 10 – (–20). This is the same as 10 + 20, which equals 30.

Subtracting a negative is the same as adding a positive: $a - (-b) = a + b$.

7) If the temperature at 8:00 a.m. is –20 degrees and the temperature at noon is –10 degrees, how much did the temperature increase?

Solution: You will need to subtract these two numbers: –10 – (–20). This is the same as –10 + 20, which equals 10.

8) If the temperature at 8:00 a.m. is 20 degrees and the temperature at noon is –10 degrees, how much did the temperature decrease?

Solution: You will need to subtract these two numbers: 20 – (–10). This is the same as 20 + 10, which equals 30.

9) If the temperature of the surface of the space shuttle two hours before launch is –20.345 degrees Celsius and the temperature at launch is 10.12 degrees Celsius, how much did the temperature increase during that two hour period?

Solution: You will need to subtract these two numbers: 10.12 – (–20.345). This is the same as 10.12 + 20.345, which equals 30.465.

10) If the temperature at 8:00 a.m. is $-20\frac{5}{8}$ degrees and the temperature at noon is $10\frac{3}{4}$ degrees, how much did the temperature increase?

Solution: You will need to subtract these two numbers: $10\frac{3}{4} - \left(-20\frac{5}{8}\right)$. This is the same as $10\frac{3}{4} + 20\frac{5}{8}$, which equals $31\frac{3}{8}$.

Helpful Hints for Adding and Subtracting Signed Numbers

SSS—If the Signs are the Same, then Sum the numbers and bring along the sign.

Example: What is –3 – 5? This could be looked at as –3 + –5. The **SIGNS** are the **SAME** (both negative), so we **SUM** (add) the numbers and bring along the sign. We should get the answer –8.

DDD—If the signs are Different, then find the Difference and take the sign of the Dominant number.

Example: What is –7 + 3? The signs are **DIFFERENT** (the 7 is negative and the 3 is positive), so we find the **DIFFERENCE** (subtract the numbers as if they are both positive). When we subtract 7 – 3, we get 4. Now we take the sign of the **DOMINANT** number (the number farthest from zero). The number farthest from zero is –7, so the answer is –4.

The Commutative Property for Addition: $a + b = b + a$

Answer the following:

1) Does $2 + 3$ equal $3 + 2$?
2) Does $2 - 3$ equal $3 - 2$?
3) Does $2 - 3$ equal $-3 + 2$?

Answers:

1) $2 + 3 = 3 + 2$. This is an example of the commutative property for addition.

   In general $a + b = b + a$. This is an important concept in Algebra. For example, we will see in Chapter 2 that $3 + 4x$ is the same as $4x + 3$.

2) $2 - 3$ does not equal $3 - 2$. This example shows that subtraction is not commutative.

   In Chapter 2 we will see that $2 - 3x$ does not equal $3x - 2$.

3) $2 - 3 = -3 + 2$. This is another example of the commutative property for addition.

   The problem $2 - 3$ can be thought of as $2 + (-3)$. We can use the commutative property for addition to write $2 + (-3)$ as $-3 + 2$.

In Chapter 2 we will see that $5 - 3x$ is the same as $-3x + 5$. This is an example of the commutative property with algebraic expressions.

Example: Answer the following:

1) $8 - 10$	2) $5 - 8$	3) $-8 + 10$	4) $-3 + (-4)$
5) $-5 - 4$	6) $7 - (-3)$	7) $-5 - (-8)$	

Solutions:

1) $8 - 10$

Thinking about it: The number 10 is bigger than 8, so we are subtracting more than 8, which means the answer will be negative. So just find $10 - 8$ and make the answer negative. $10 - 8$ is 2, so the answer is -2. This is similar to having \$8 and wanting to buy something that costs \$10. You don't have enough money. You are \$2 lower than the amount needed.

Using SSS or DDD: $8 - 10$ is the same as $8 + (-10)$. The 8 is positive and the 10 is negative, so the signs are different. Using DDD, we know that if the signs are different, find the difference (subtract) and use the sign of the dominant number (the number furthest from 0). Noticing that the signs are different, we subtract to get 2 and use the sign of the dominant number, (-10), to get -2.

Using the commutative property: $8 - 10$ is the same as $-10 + 8$. We can see the signs are different, so find the difference, which is 2, and take the sign of the dominant number to get -2.

2) $5 - 8$

Thinking about it: The number 8 is bigger than 5, so we are subtracting more than 5, which means the answer will be negative. Just find $8 - 5$ and make the answer negative. $8 - 5$ is 3, so the answer is -3. This is similar to having \$5 and wanting to buy something that costs \$8. You don't have enough money. You are \$3 lower than the amount needed.

Using SSS or DDD: $5 - 8$ is the same as $5 + (-8)$. The 5 is positive and the 8 is negative, so the signs are different. Using DDD, we know that if the signs are different, find the difference (subtract) and use the sign of the dominant number (the number furthest from 0). Noticing that the signs are different, we subtract to get 3 and use the sign of the dominant number, (-8), to get -3.

Using the commutative property: $5 - 8$ is the same as $-8 + 5$. We can see the signs are different, so find the difference, which is 3, and take the sign of the dominant number to get -3.

3) $-8 + 10$

Thinking about it: The number 10 is bigger than 8, so we are adding on more than -8, which means the answer will be positive. So just find $10 - 8$, so the answer is 2. This is similar to being in debt \$8 and then earning \$10. You pay off your debt of \$8, and you still have \$2.

Using SSS or DDD: The signs are different. Using DDD, we know that if the signs are different, find the difference (subtract) and use the sign of the dominant number (the number furthest from 0). Noticing that the signs are different, we subtract to get 8 and use the sign of the dominant number, (10), to get 2.

Using the commutative property: $-8 + 10$ is the same as $10 - 8$ which equals 2.

4) $-3 + (-4)$

Thinking about it: We are at -3 and then we are adding on -4. This is similar to having a debt of \$3 and then adding onto that debt 4 more dollars. We would then be \$7 in debt, or in other words -7.

Using SSS or DDD: The signs are the same (both negative). Using SSS, we know that if the signs are the same, sum them (add) and bring along the sign to get -7.

Using the commutative property: $-3 + (-4)$ is the same as $-4 + (-3)$ which equals -7.

5) $-5 - 4$

Thinking about it: We are at -5 and then we are subtracting 4. This is similar to having a debt of \$5 and then adding onto that debt 4 more dollars. We would then be \$9 in debt, or in other words -9.

Using SSS or DDD: $-5 - 4$ is the same as $-5 + (-4)$. The signs are the same (both negative). Using SSS, we know that if the signs are the same, sum them (add) and bring along the sign to get -9.

Using the commutative property: $-5 + (-4)$ is the same as $-4 + (-5)$ which equals -9.

6) $7 - (-3)$

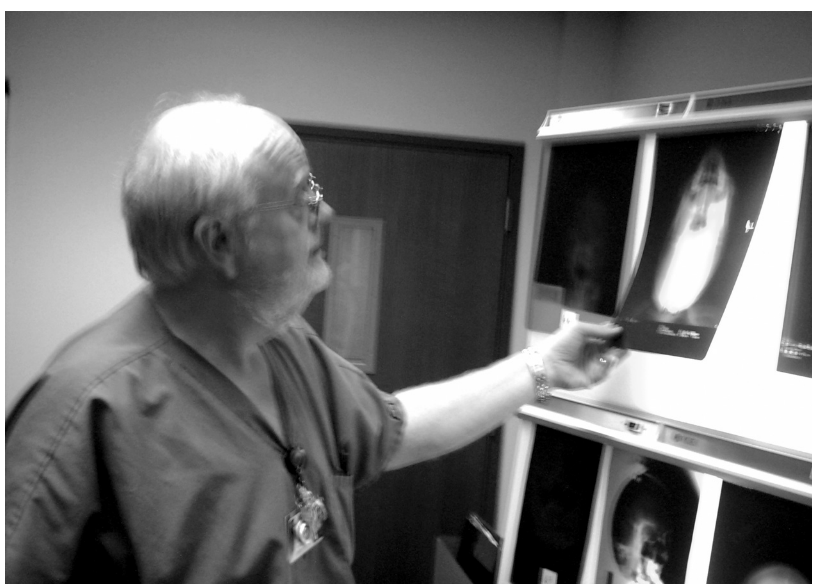
We first have to use the fact that subtracting a negative is the same as adding a positive. So $7 - (-3)$ is the same as $7 + 3$ which equals 10.

7) $-5 - (-8)$

We first have to use the fact that subtracting a negative is the same as adding a positive. So $-5 - (-8)$ is the same as $-5 + 8$ which equals 3.

You Try These #1

X-ray film processors are monitored daily to ensure consistency in development of films. The processor should be tested at a time during its active use, using freshly sensitized sensitometric control strips. Three parameters tested are the contrast, fog, and developer temperature. Day One the processor contrast is 0.10, the processor fog measures –0.05, and the standard temperature is 98 degrees.

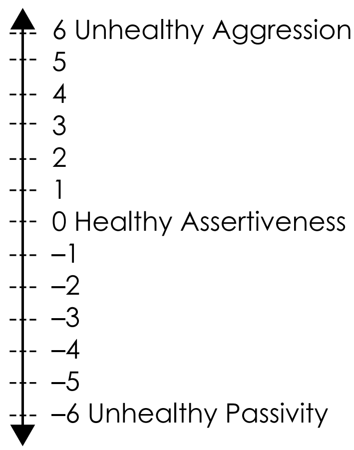


If the processor contrast drops from 0.10 to –0.15, what is the difference in processor contrast?

If the fog rises from –0.05 to 0.07, what is the difference in processor fog?

Example: Human Services—Passivity to Aggression Scale

Below is the passivity to aggression scale. The lower a person is on the vertical scale, the more passive the person tends to be. The higher a person is on the vertical scale, the more aggressive the person tends to be. The closer to the middle a person is on the scale, the closer the person is to healthy assertiveness.



A person may start with an unhealthy passivity, and then there may be an event (perhaps even a small event) which may cause a rapid change in the person's behavior from unhealthy passivity to unhealthy aggression. The person may then go from an unhealthy aggression level to a less severe passivity. The process continues over time with smaller and smaller changes until the person's behavior levels off to a healthy assertiveness level.

For example, let's say a person is at a level of –6 (very unhealthy passivity). An event occurs which triggers the person to go up 10 on the scale. What would be that person's new value on the scale? ***Answer:*** 4

Write the previous problem with numbers. ***Answer:*** **$-6 + 10 = 4$.**

You Try These #2

2a) A person is currently at 4 (on the passivity aggression scale). The person goes down 7 units. What is that person's value on the scale? Write the previous problem with numbers.

2b) A person is currently at –2 (on the passivity aggression scale). The person goes down 7 units. What is his new value on the scale? Write the previous problem with numbers.

2c) A person is currently at –8 (on the passivity aggression scale). The person goes up 7 units. What is her new value on the scale? Write the previous problem with numbers.

1.2 Signed Numbers Worksheet 1

Add or subtract as indicated.

1) $-19 - 6$

2) $-22 + (-11)$

3) $17 + (-12)$

4) $7 - 23$

5) $-13 - (-5)$

6) $-8 - 14$

7) $-6 + (-9)$

8) $-13 + 8$

9) $14 - 19$

10) $-8 - (-12)$

11) $16 + (-11)$

12) $-7 - 9$

13) $4 - (-9)$

14) $-3 - (-7)$

15) $5 - (-3)$

1.2 Signed Numbers Worksheet 2

Perform the indicated operations.

1) $-9 + 39$

2) $-12 + (-28)$

3) $-4 - 15$

4) $-6 - (-11)$

5) $4 - (-9)$

6) $-10 + (-12)$

7) $-3 - (-7)$

8) $-15 - 17$

9) $23 + 40$

10) $-8 + (-15)$

11) $-6 - (-14)$

12) $7 - (-13)$

13) $-16 + 21$

14) $-30 - 21$

15) $44 - (-19)$

16) $55 + (-35)$

17) $-71 + (-29)$

18) $-61 - (-46)$

19) $87 - (-58)$

20) $-76 + 17$

21) $56 - (-62)$

22) $-29 - (-37)$

1.2 Exercise Set

Solve these numerical problems.

1) $7 - 5$

2) $5 - 7$

3) $7 - 10$

4) $10 - 7$

5) $-7 + 5$

6) $7 - 5$

7) $-7 - 10$

8) $-5 - 7$

9) $-8 + 3 - 10$

10) $5 - 8 - 10 + 2$

11) $-1.23 + 4.5$

12) $1.26 - 0.021$

13) $-0.12 - 5.12$

14) $-3.4 - 3.4$

15) $-\frac{1}{2} - 5\frac{3}{4}$

16) $-3\frac{1}{8} + \frac{5}{6}$

17) Rank the following numbers from lowest to highest:

$-3,\ \frac{0}{5},\ \frac{-1}{2},\ \frac{-1}{3},\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{6},\ 0.66,\ 0.\overline{6},\ 0.2\overline{3},\ 0.\overline{23}$

18) Rank the following numbers from lowest to highest:

$-0.3,\ \frac{5}{0},\ -7\frac{2}{3},\ -7.66,\ -7.76,\ -6,\ 6,\ 7.6,\ 6.7,\ 6.07,\ 7.\overline{6}$

19) Rank the following numbers from lowest to highest:

$-2,\ \frac{0}{3},\ -6\frac{1}{3},\ -6.33,\ -6.3,\ -6,\ -6.03,\ -6.\overline{3},\ \frac{-1}{6},\ \frac{-1}{4},\ \frac{1}{4},\ \frac{1}{3},\ \frac{1}{6},\ 0.16,\ 0.\overline{16},\ 0.1\overline{6}$

20) Rank the following numbers from lowest to highest:

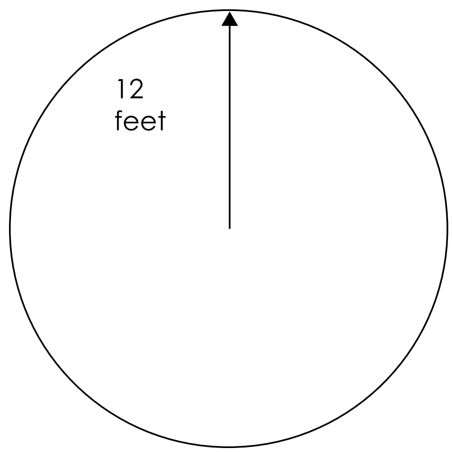
$-2, 2, \frac{0}{3}, -6\frac{1}{3}, -6.33, -6.3, -6, -6.03, -6.\overline{3}, 6.33, 6.3, 6, 6.03, 6.\overline{3}$

21) A patient with an injury to the right elbow has active range of motion from 30 to 90 degrees. What is the total active range of motion for the elbow?

22) A patient with an injury to the left knee has active range of motion equal to –10 to 140 degrees. What is the total active range of motion for this patient?

23a) A patient ambulated in a circle around the Physical Therapy Department. The distance between the center of the circle and a point where the patient walked is 12 feet (Figure 1.1). Calculate the total distance the patient walked.

FIGURE 1.2 DISTANCE = $2\pi r$



23b) The patient had a goal to ambulate 50 feet. Did the patient meet her goal?

1.3 Multiplication and Division of Signed Numbers

When you are *multiplying or dividing*, an even number of negative signs will make the answer positive and an odd number of negative signs will make the answer negative.

Think of the meaning of this sentence: "I DON'T have NO money." Two negatives make a positive. So, when someone says, "I DON'T have NO money," then they must have some money.

How about the meaning of this sentence: "Turn the off switch off." If you turn the off switch off, then you must be turning it on. This is another example of how two negatives make a positive when you multiply or divide.

Example 1: Simplify $-2(-3) - 5$

Solution: First, we need to realize that the operation going on between the -2 and the (-3) is multiplication. Consequently, the expression $-2(-3) - 5$ is the same as $-2 \times -3 - 5$. Next, we need to multiply -2×-3 before we do the subtraction. We will see much more of the order of operation in the next section, but for now, you need to know that you do multiplication and division before you do addition and subtraction. We get that -2×-3 is 6, so we have $6 - 5$, and this is equal to 1.

Example 2: *Method 1:* Simplify $\frac{-2 - 3(-4)}{-10 + 12}$

Solution: First, the expression $\frac{-2 - 3(-4)}{-10 + 12}$ is the same as $\frac{-2 + (-3) \times (-4)}{-10 + 12}$. Multiply -3 times -4 in the numerator to get $\frac{-2 + 12}{-10 + 12}$. Now perform the addition in the numerator and denominator to get $\frac{10}{2} \rightarrow 5$. Note: The symbol $\rightarrow$ means "yields" or "gives us."

Example 2: *Method 2:* We could have also looked at the problem $\frac{-2 - 3(-4)}{-10 + 12}$ as $\frac{-2 - (3) \times (-4)}{-10 + 12}$. Since $(3) \times (-4) \rightarrow (-12)$, this gives us $\frac{-2 - (-12)}{-10 + 12}$. Next, we need to realize that $-(-12)$ is the same as $-1 \times (-12)$, which is 12. This gives us $\frac{-2 + 12}{-10 + 12}$. Now perform the addition in the numerator and denominator to get $\frac{10}{2} \rightarrow 5$.

The Commutative Property for Multiplication: $a \times b = b \times a$

Answer the following:

1) Does $(-2)(3)$ equal $3(-2)$?

2) Does $(2)(-3)$ equal $(-3)(2)$?

3) Does $(-2)(-3)$ equal $(-3)(-2)$?

4) Does $4 + 2 \times -3$ equal $4 + (-3) \times 2$?

Answers:

1) Yes. $(-2)(3) = -6$ and $3(-2) = -6$
2) Yes. $(2)(-3) = -6$ and $(-3)(2) = -6$
3) Yes. $(-2)(-3) = 6$ and $(-3)(-2) = 6$
4) Yes. $4 + 2 \times -3$ is $4 + (-6)$ which equals -2. $4 + (-3) \times 2$ is $4 + (-6)$ which equals -2.

You Try These

1) $-(-3)$

2) $5 + (-3)$

3) $-(-(-3))$

4) $-2 - (-3) + (-2)$

5) $\dfrac{-2 + (-3) - (-5)}{3 - 4(5)}$

6) $7(-3)$

7) $(-7)(3)$

8) $(-7)(-3)$

9) $\dfrac{4}{-8}$

10) $\dfrac{-4}{8}$

11) $\dfrac{-4}{-8}$

12) $-(-7)$

13) $-(-(-7))$

14) $-(-3)(-5)$

15) $(-2)(-4)(5)(-3)$

16) $\left(\dfrac{-1}{2}\right)\left(\dfrac{-3}{4}\right)$

17) $\left(5\frac{1}{8}\right)\left(-3\frac{5}{8}\right)$

18) $\left(\dfrac{3}{4}\right)\left(\dfrac{-1}{2}\right)$

19) $-3\frac{1}{4} \div 5\frac{1}{8}$

20) $\dfrac{(-3)(-4)}{(-2)(5)}$

Irrational Numbers

An **irrational** number is any number which is not rational (the decimal goes on forever and does not repeat).

Is $\sqrt{25}$ rational? What does $\sqrt{25}$ equal? How could you check your answer?

Is $\sqrt{2}$ rational?

The union of the set of **rational numbers** and **irrational numbers** is called the set of **real numbers**.

The set of integers is a subset of the set of rational numbers.

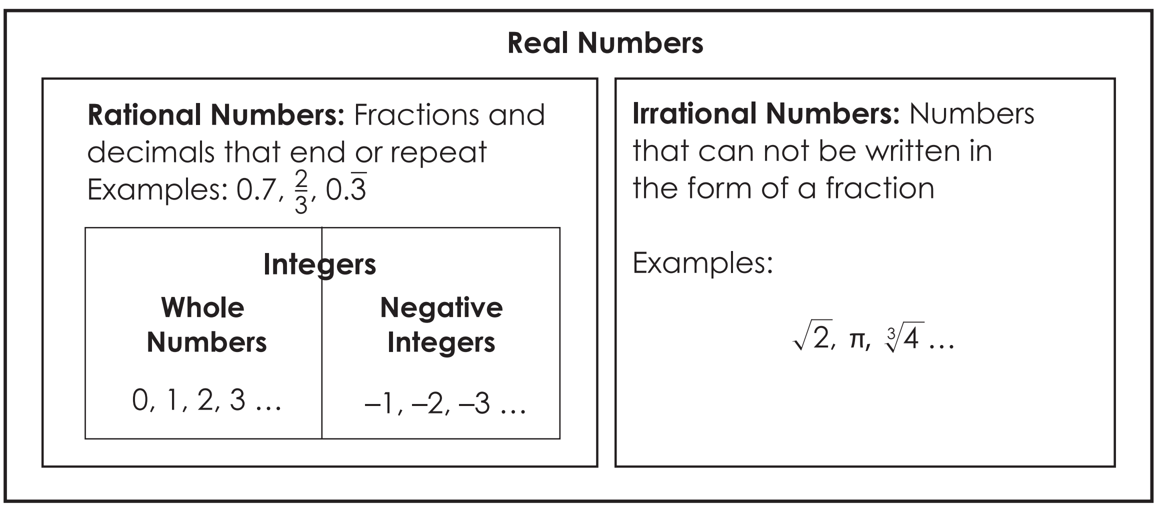
The set of integers is a subset of the set of real numbers.

The set of rational numbers is a subset of the set of real numbers.

The set of irrational numbers is a subset of the set of real numbers.

The set of integers is not a subset of the set of irrational numbers.

FIGURE 1.3 REAL NUMBERS



1.3 Exercise Set

Answer the following numerical problems:

1) $\frac{-5 - 2}{-3 + 5}$

2) Does $\frac{-2}{3} = \frac{2}{-3}$?

3) Does $\frac{-2}{3} = -\frac{2}{3}$?

4) Does $\frac{-2}{3} = \frac{-2}{-3}$?

5) $5 - (-2)(-3)$

6) $-7 - (-8)$

7) $-7(-8)$

8) $(-2)(-3) - (2 - 5)$

9) $-5 - (-7)$

10) $\frac{-8 - 2}{-15 + 5}$

11) $\frac{(-8) - 2}{(-15) + 5}$

12) $\frac{(-8) - (2)}{(-15) + (5)}$

13) $\frac{(-8 - 2)}{(-15 + 5)}$

14) $\frac{(-8)(-2)}{(-15)(5)}$

15) $\frac{-8(-2)}{-15(5)}$

16) $\frac{-(8 - 2)}{-(15 + 5)}$

17) $\frac{-(2 - 8)}{-(-15 + 5)}$

18) $\frac{-(8) - (2)}{-(15) + (5)}$

19) $\frac{-8 - 2(-10)}{-15(2) + 5}$

20) $\frac{-8}{-15 - 5}$

1.4 Order of Operations, Exponents, and Roots

PEMDAS (Please Excuse My Dear Aunt Sally) is a way to remember the order of operations.

P = Parentheses: Do any operation inside of parentheses, brackets, or any other grouping symbol.

E = Exponents: Raise numbers to the exponent.

MD = Multiplication and Division: Multiply and Divide. Do these as they appear from left to right.

AS = Addition and Subtraction

Example 1: Simplify $5 - 4 \times 5$

Solution: Do the multiplication first and we get $5 - 20$, which gives us -15

Example 2a: Simplify -3^2

Solution: The only thing raised to the second power (squared) is the 3. The answer is -9.

Example 2b: Simplify $(-3)^2$

Solution: The power of 2 affects everything inside the parentheses. The answer is 9.

Example 2c: Simplify $-(3)^2$

Solution: The only thing raised to the second power (squared) is the 3. The answer is -9.

Example 2d: Simplify $-(3^2)$

Solution: The only thing raised to the second power (squared) is the 3. The answer is -9.

Example 3a: Simplify $5 - 3^2$

Solution: Square the 3 and we get $5 - 9$. The answer is -4.

Example 3b: Simplify $5 + (-3)^2$

Solution: Square the -3 and we get $5 + 9$. The answer is 14.

Example 3c: Simplify $(5 - 3)^2$

Solution: First do what is inside the parentheses to get 2^2. The answer is 4.

Example 4a: Simplify $(2 - 3) \times [(2 - 5)^2 \times -5^2]$

Solution: When you have grouping symbols, such as parenthesis or brackets, inside other grouping symbols, do the inner most grouping operation first. Since the $(2 - 5)$ is the inner most grouping symbol for the following example, we do this first and we get

$$(2 - 3) \times [(-3)^2 \times -5^2].$$

Now follow the order of operations inside the brackets and we get

$$(2 - 3) \times [9 \times -25].$$

Simplifying the parenthesis and the brackets we get

$$-1 \times -225, \text{ which gives us } 225.$$

Example 4b: Simplify $8 \div 2 \times 4$

Solution: Many people get this problem incorrect because they think that multiplication is performed before division. However, this is not true. You do multiplication and division in the order in which they appear from left to right. Therefore, on this problem we do the division first and we get

$$8 \div 2 \times 4$$

$$4 \times 4 \rightarrow 16.$$

Example 4c: Simplify $8 \div (2 \times 4)$

Solution: Since there is a grouping symbol around the 2×4, we do the multiplication first. We get

$$8 \div (2 \times 4)$$

$$8 \div 8 \rightarrow 1.$$

Example 4d: Simplify $8 \times 2 \div 4$

Solution: On this problem we do the multiplication first because multiplication is listed before the division. We get

$$8 \times 2 \div 4 \rightarrow 16 \div 4 \rightarrow 4.$$

Example 4e: Simplify $125 - 3[(27 - 38) - (20 - 5^2)]$

Solution: Do the operations in the inner sets of parenthesis first and we get

$$125 - 3[(-11) - (20 - 25)]$$

$$125 - 3[-11 - (-5)]$$

$$125 - 3[-11 + 5]$$

$$125 - 3(-6).$$

This gives us $125 + 18$ which equals 143.

You Try These

1) $6 - 3 \times 5$

2) $6 \div 3 \times 2$

3) 4^2

4) 5^3

5) 2^4

6) $(6 - 3)^2$

7) $6^2 - 3^2$

8) $(-3)^2$

9) -3^2

10) $-3(6 \times (3 - 5))$

11) $2(3)^2$

12) $(2 \times 3)^2$

13) $2^2 \times 3^2$

14) $(6 - (-3)^2)^2$

1.4 Exercise Set

1. $\dfrac{-3 - 2(-5)^2}{-1 - (-2)(-6)}$

2. $-6 \div -3 \times 2$

3. Answer the following numerical problems:

a) $(-2)(-7)$

b) $-2(-7)$

c) $(-2) - 7$

d) $(-2) - (-7)$

e) $-(-2) + (-7)$

f) $(-2)^2$

g) -2^2

h) $-3 - 5$

i) $-32 + 15$

j) $15 - 32$

k) $-32 - 15$

l) $-32 - (-15)$

m) $-32 + (-15)$

n) $(-32)(-15)$

o) $(-3)(-5)(2)$

p) $(-3)(-5)(-2)$

q) $(-3)(-2)^3$

r) $(-3 \times -2)^3$

s) $(-3)^3(-2)^3$

t) 2×3^3

u) $-6 \div 3 \times 2$

v) $3 - 2 \times 4$

w) $-3 - 3 + 2(-3)$

x) $-2 + 5 - 7 + 5 - 3 + 10 - 5$

y) $\dfrac{-3}{-6} - \dfrac{-10 - (-3)}{-5(3) - 4(-2)}$

z) $\dfrac{2 - 3(-2)(-4)}{5 + (-2)(-3)}$

1.5 Evaluating Algebraic Expressions

A formula used by radiologists is **voltage** (V) equals **amperage** (I) times **resistance** (R) or $V = IR$. **Note**: IR means $I \times R$. If the Resistance is 10 ohms, we have the equation $V = I \times 10$. Or $V = 10I$. Complete the table below.

Amperage (I)	$V = 10I$
0	$V = 10(0) \rightarrow 0$
15	$V = 10(15) \rightarrow 150$
20	
25	

Solution: When $I = 20$, we get $V = 10(20) \rightarrow 200$. When $I = 25$, we get $V = 10(25) \rightarrow 250$.

Example 1a: Complete the table below for the equation $y = -2x + 3$.

x	$y = -2x + 3$
-2	$y = -2(-2) + 3$ $y = 4 + 3$ $y = 7$
-1	
0	
1	$y = -2(1) + 3$ $y = -2 + 3$ $y = 1$
2	

Example 1a: Complete the table for the equation $A = \frac{1}{2}bh$. Note: This is the formula for the area of a triangle, where A is the area and b is the base and h is the height.

b	h	$A = \frac{1}{2}bh$
0	10	$A = \frac{1}{2}(0)(10)$ $A = 0$
10	12	
20	14	
30	16	$A = \frac{1}{2}(30)(16)$ $A = 240$
40	18	

Example 2: Evaluate $\frac{x - y}{x + 5} + \frac{x + y}{x - 4}$ if $x = 10$ and $y = 7$.

When evaluating expressions or equations remember to put parentheses around the numbers you are substituting in the expressions.

Substituting the 10 for x and 7 for y, we get

$$\frac{(10) - (7)}{(10) + 5} + \frac{(10) + (7)}{(10) - 4}$$

$$\frac{10 - 7}{10 + 5} + \frac{10 + 7}{10 - 4}$$

$$\frac{3}{15} + \frac{17}{6}$$

$$\frac{1}{5} + \frac{17}{6}$$

$$\frac{6}{30} + \frac{85}{30}$$

$$\frac{91}{30} = 3\frac{1}{30}.$$

Example 3: The number of calories burned running on a treadmill can be calculated by the equation $c = 0.003p(4.5m + 100g)t$, where c is the number of calories burned, p is the pounds that you weigh, m is the miles per hour you run, g is the grade (steepness) of the treadmill and t is the number of minutes you run. Complete the table below.

p	m	g	t	c
130 pounds	4 miles/hour	2% grade	30 minutes	You should get 234.
150 pounds	4 miles/hour	2% grade	30 minutes	
150 pounds	6 miles/hour	3% grade	20 minutes	

Solution:

1) $n = 0.003 \times 130(4.5 \times 4 + 100 \times 0.02) \times 30$

$n = 0.39(18 + 2)30$

$n = 0.39 \times 20 \times 30$

$n = 234$

2) $n = 0.003 \times 150(4.5 \times 4 + 100 \times 0.02) \times 30$

$n = 0.45(18 + 2)30$

$n = 0.45 \times 20 \times 30$

$n = 270$

3) $n = 0.003 \times 150(4.5 \times 6 + 100 \times 0.03) \times 20$

$n = 0.45(27 + 3)20$

$n = 0.45 \times 30 \times 20$

$n = 270$

You Try These

1) Evaluate $\frac{x-1}{2-y} + \frac{x}{y}$ when $x = -3$ and $y = -2$.

2) The Fahrenheit temperature of a person experiencing hyperthermia can be modeled by the linear equation $F = 0.36t + 99$, where F is the person's temperature and t is the amount of time a person has been experiencing hyperthermia (in units of minutes).

   a) Complete the table for the linear relationship between the amount of time a person has been experiencing hyperthermia and their Fahrenheit temperature.

Time (t) a person has been experiencing hyperthermia (in units of minutes)	**Fahrenheit temperature $F = 0.36t + 99$**
0	$F = 0.36(0) + 99$. So $F = 99$
1	$F = 0.36(1) + 99$. So $F = 99.36$
2	$F = 0.36(2) + 99$. So $F = 99.72$
3	
4	
10	

   b) According to your table, for each additional minute of time a person is experiencing hyperthermia, how much is their Fahrenheit temperature increasing?

3) The following linear model can be used to approximate a person's *percent saturation of hemoglobin* (H) given their PO_2 level (P). The linear model is $H = 2P - 6$.

   a) Use this linear model to complete the following table:

PO_2	**Predicted hemoglobin level based on the model $H = 2P - 6$**	**Actual hemoglobin level (Note: An equation does not usually give you actual levels.)**
10	$H = 2(10) - 6$. So $H = 14$	14
20	$H = 2(20) - 6$. So $H = 34$	35
30		57
40		75
50		85
60		90
70		93

   b) In your opinion, for what PO_2 levels does your model give a good approximation for the actual hemoglobin level? Explain your answer in a sentence or two.

4a) The average cost (C) of making (n) number of boxes of syringes can be described by the equation $C = -0.4n + 10$. Complete the following table:

(Number of boxes of syringes)	Cost
0	$C = -0.4(0) + 10 \rightarrow 10$
1	$C = -0.4(1) + 10 \rightarrow 9.6$
2	
10	

4b) How many boxes of syringes must be produced to reduce the average cost to 0? ***Note:*** You will need to substitute more values into the equation.

5a) The cost of renting a building is dependent on the square footage of the building and can be modeled by the linear equation $C = 2A + 300$, where C is the cost of the rent per month and A is the square footage (floor area) for the building. Complete the following table:

Area	Cost
500	$C = 2(500) + 300 \rightarrow 1000 + 300 \rightarrow 1300$
1000	$C = 2(1000) + 300 \rightarrow 2000 + 300 \rightarrow 2300$
1500	
2000	
2500	

5b) Use the table to find approximately what area will have a cost of $2000? ***Hint:*** Using the table above you will know that it is between 500 and 1000. Now try values in between to narrow in on the answer.

6a) Evaluate $-2a^2 - 3ab + 4b^2$ if $a = -3$ and $b = -2$.

Solution: First, substitute in the values for x and y with parentheses around them, and we get $-2(-3)^2 - 3(-3)(-2) + 4(-2)^2$. Following the order of operations, we now do exponents to get $-2(9) - 3(-3)(-2) + 4(4)$. Now, doing the multiplication, we get $-18 - 18 + 16$. Now, adding the -18 with the -18, we get -36 and $-36 + 16$, which equals -20.

6b) Evaluate $-2a^2 - 3ab + 4b^3$ if $a = -2$ and $b = -1$.

6c) Evaluate $-2a^2 - 3ab + 4b^2$ if $a = 3$ and $b = -2$.

6d) Evaluate $-2a^2 - 3ab + 4b^2$ if $a = -3$ and $b = 2$.

6e) Evaluate $-2a^2 - 3ab - 4b^2$ if $a = -3$ and $b = -2$.

7a) Evaluate $\dfrac{-2x^2 - 3y}{-x - 2y}$ if $x = -2$ and $y = -3$.

Solution: Substituting our values for x and y, we get $\dfrac{-2(-2)^2 - 3(-3)}{-(-2) - 2(-3)}$. Raising the -2 to the second power, we get $\dfrac{-2(4) - 3(-3)}{-(-2) - 2(-3)}$. Now multiplying, we get $\dfrac{-8 + 9}{2 + 6}$. Now adding, we get $\dfrac{1}{8}$.

7b) Evaluate $\dfrac{-2x^3 - 5y}{-x + 2y}$ if $x = -4$ and $y = -2$.

Solution: Substituting our values for x and y, we get $\dfrac{-2(-4)^3 - 5(-2)}{-(-4) + 2(-2)}$. Raising the -4 to the third power, we get $\dfrac{-2(-64) - 5(-2)}{-(-4) + 2(-2)}$. Now multiplying, we get $\dfrac{128 + 10}{4 - 4}$. Now adding, we get $\dfrac{138}{0}$. Division by 0 is undefined. So the answer is undefined.

7c) Evaluate $\dfrac{-x + y}{-x - y}$ if $x = -5$ and $y = -4$.

1.5 Exercise Set

1) Evaluate $\frac{x+3}{2x-7}+\frac{y-x}{2x-3y}$ when $x=-4$ and $y=2$.

2) Evaluate $\frac{2x-3}{2x-7}+\frac{x-y}{x-3y}$ when $x=-4$ and $y=-2$.

3) Evaluate $\frac{x-y}{2x-y}+\frac{y-x}{3(x-y)}$ when $x=-3$ and $y=-2$.

4) Evaluate $\frac{x-3y}{2x-7}+\frac{-2x-y}{x-3y}$ when $x=-3$ and $y=-2$.

5a) Complete the table for the Fahrenheit to Celsius conversion formula $c=\frac{5}{9}(f-32)$.

Fahrenheit	**Celsius**
0	
1	
2	
3	
4	
5	
6	

5b) How much is the Celsius temperature increasing for each increase of 1 degree in Fahrenheit?

6) The following data was gathered on the year the Indy 500 was run and the winning times:

Year	Winning time in minutes
1920	338
1930	298
1940	262
1950	rained out
1960	216
1970	192
1980	209
1990	161

The winning times of the Indy 500 can be modeled (predicted) by the linear equation **$W = -2.3t + 321$**, where W is the winning time and t is the number of years since the inception of the race in 1920.

a) Complete the table for the linear relationship between the winning times and the year the race was run.

t measured in years ($t = 0$ is 1920)	**Winning time from the equation $W = -2.3t + 321$**
0	
10	
20	
30	
40	
50	
60	
70	
80	

b) According to your table, for each year, what is happening to the winning times?

c) The equation $W = -2.3t + 321$ is called a "**model**" because the equation models or predicts values for this situation. Does a model give you the actual data value for a given year?

d) Does the model $W = -2.3t + 321$ give values close to the actual data values (data points)?

If a model gives values that are close to the actual data points, then it is a good model for the situation.

7a) Evaluate $-3a^2 - 2ab - b^2$ if $a = -3$ and $b = -2$.

7b) Evaluate $-3a^2 - 2ab - b^2$ if $a = -3$ and $b = 2$.

7c) Evaluate $-3a^2 + 2ab - b^2$ if $a = -3$ and $b = -2$.

7d) Evaluate $3a^3 - 2ab - b^2$ if $a = -3$ and $b = -2$.

7e) Evaluate $-3a^2 - 2ab^2 - b^3$ if $a = -3$ and $b = -2$.

Review Problems for Review Chapter and Chapter 1

1) A new hospital is under construction. The medical professionals must communicate with the contractors to make sure that the building's interior satisfies the medical needs of the patients. The nursing staff tells the contractor that in order for a rectangular hospital room to have enough wall space for two beds, two chairs, a door, and all of the medical equipment, the rooms must have a perimeter of at least 40 meters. Due to the length of the beds and needing walking space in front of the beds, the width of the rooms must be at least 5 meters. What would be the maximum length of a rectangular hospital room under these constraints?

2) Compute the area of the rectanglar hospital room described in problem number 1.

3) A nurse needs to give a patient $\frac{1}{4}$ of a mg of *Demerol* in 3 equal doses. What fraction of a milligram does the patient get per dose?

4) The medication *Lanoxin* is prescribed for an adult as 0.25 mg. The children's dose is 20% of the adult amount. How much is the children's dose? Give your answer as both a decimal and a fraction.

5) The medication *Lanoxin* is prescribed for an adult as 0.25 mg. The children's dose is 20% of the adult amount. How much more is the adult dose than the children's dose? Give your answer as both a percent and a fraction.

6) A patient is prescribed 12.5 mg of *Librium* every 4 hours. How many mg does the patient get in a 24 hour day?

7) A nurse needs to give a patient 0.1 mg in 4 equal doses. What fraction of a milligram does the patient get per dose?

8) The medication, *Benadryl*, for an adult is prescribed as $\frac{1}{2}$ tbs. The children's dose is $\frac{1}{8}$ of the adult amount. How much is the children's dose? Give your answer as both a fraction and a decimal.

9) The medication, *Dilantin*, for an adult is prescribed as $\frac{1}{2}$ tbs. The children's dose is 25% of the adult amount. How much more is the adult dose than the children's dose? Give your answer as both a percent and a fraction.

10) The temperature of a patient is 100.2. The normal body temperature is 98.6. How much above normal is the patient's temperature?

11) For the following problems write a numerical expression and solve.

a) At 5:00 the temperature is –2 degrees outside. Over the next hour the temperature dropped 5 degrees. What is the temperature at 6:00?

b) At 5:00 the temperature is 2 degrees outside. Over the next hour the temperature dropped 5 degrees. What is the temperature at 6:00?

c) At 5:00 the temperature is –5 degrees outside. Over the next hour the temperature rose 7 degrees. What is the temperature at 6:00?

d) At 5:00 the temperature is –20 degrees outside. Over the next hour the temperature rose 7 degrees. What is the temperature at 6:00?

e) At 5:00 the temperature is –2.34 degrees outside. Over the next hour the temperature dropped 5.9 degrees. What is the temperature at 6:00?

f) At 5:00 the temperature is –2.5 degrees outside. Over the next hour the temperature rose 5.1 degrees. What is the temperature at 6:00?

g) A new born baby is 18 inches tall, her twin sister is 0.3 inches shorter. How tall is her sister?

h) A patient's temperature at 5:00 is 100.8. Over the next hour his temperature rises 1.2 degrees. What is his temperature at 6:00?

i) A patient's temperature at 5:00 is 100.8. Over the next hour his temperature decreases 1.2 degrees. What is his temperature at 6:00?

j) A patient's temperature at 5:00 is 100.8. Over the next hour his temperature decreases –1.2 degrees. What is his temperature at 6:00?

k) A patient's temperature at 5:00 is 100.8. His temperature rises at the rate of 1.2 degrees every hour. What would be his temperature at 9:00?

12) A patient's temperature at 5:00 is 100.8. His temperature rises at the rate of 1.2 degrees every hour.

a) Use this information to complete the following table:

Time	Fahrenheit Temperature
5:00	
6:00	
7:00	
8:00	

b) Write an equation for the above situation using (t) for the time (let the starting time 5:00 be $t = 0$) and (f) for the person's Fahrenheit temperature.

13a) The number of cases of hepatitis B (in units of thousands) in the United States for the years 1990 to 1995 can be modeled by the equation $n = -1.8t + 18$, where n is the number of cases of hepatitis B (in units of thousands) and t is the number of years since 1990. Complete the following table:

t(t = 0 is 1990)	$n = -1.8t + 18$
0	
1	
2	
3	
4	

b) Dosage calculations for children are sometimes based on body surface area (BSA). BSA is measured in units of m^2 (square meters). The formula to calculate body surface area (BSA) is $\text{BSA} = \frac{4W + 7}{W + 90}$, where W is the child's weight in kilograms. Complete the table.

W	BSA
2	
10	
20	

c) Evaluate the expression $\frac{4W + 7}{W + 90}$ when $W = -90$.

d) Evaluate the expression $\frac{4W + 7}{W + 90}$ when $W = -100$.

e) Evaluate the expression $\frac{4W + 7}{W + 90}$ when $W = -80$.

14) The amount of money spent annually for physicians' services in the United States from 1960 to 1995 is approximated by the model, $Amount = \frac{t^2}{4} - \frac{13t}{4} + 15$, where the amount spent is in billions of dollars and t represents the year, with $t = 0$ corresponding to 1960. Complete the following table:

Year	Amount
1950 $t = -4$	
1960 $t = 0$	
1964 $t = 4$	

15) A new hospital is under construction. The medical professionals must communicate with the contractors to make sure that the building's interior satisfies the medical needs of the patients. The nursing staff tells the contractor that in order for a rectangular hospital room to have enough wall space for two beds, two chairs, a door, and all of the medical equipment, the rooms must be $4\frac{1}{2}$ meters wide by $5\frac{1}{4}$ meters long.

a) Compute the area of the rectangular hospital room.

b) Compute the perimeter of the hospital room described in problem number 1.

16 a) 150 cm is equal to how many mm?

b) 150 cm is equal to how many meters?

c) 1500 mg is equal to how many gm?

d) 500 cc is equal to how many mL?

e) What metric unit is used to measure the volume of a liquid? Circle one.

Meters Grams Liters

17a) Evaluate $\frac{-3^2 - (-4)^2}{5 - 7}$.

17b) Evaluate $-3 - (-5)(-3)$.